Long shadows of financial shocks: an endogenous growth perspective*

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Abstract

The Great Recession has resulted in a seemingly permanent level shift in many macroeconomic variables. This paper presents a microfounded general equilibrium model featuring frictional labor markets and financial frictions that generates procyclical R&D expenditures and replicates business cycle features of establishment dynamics. This allows demonstrating the channels through which productivity and financial shocks influence the aggregate endogenous growth rate of the economy, creating level shifts in its balanced growth path.

I find that financial shocks are an important driver of the aggregate fluctuations and their influence is especially pronounced for establishment entry. Since the growth rate of the economy can in principle be affected by policy measures, I examine the macroeconomic and welfare effects of applying several subsidy schemes.

Keywords: business cycles, establishment dynamics, endogenous growth, working capital, financial shocks

JEL codes: E32, G01, J63, J64, O3, O40

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1 Introduction

The experience of the Great Recession and its aftermath has compelled many macroeconomists to examine the links between the financial sector and real activity. This paper presents a model of heterogeneous, monopolistically competitive establishments who endogenously choose the intensity of research and development. The model features also endogenous entry and exit, and incorporates search and matching frictions in the labor market, as well as a reduced form of the financial shock. The paper brings together several strands of literature on business cycles, firm dynamics and endogenous growth and carries important policy implications for industrial policy over the business cycle.

The two main mechanisms that generate volatile and procyclical R&D expenditures are increased willingness of incumbents to invest in R&D in good times, as well as procyclical entry rates. This translates to the endogenous growth rate of the economy to be also procyclical, and gives rise to hysteresis effects, as in response to transitory shocks the balanced growth path permanently shifts. As a consequence, welfare effects of business cycle are much higher than for the exogenous growth models, as consumption is not only volatile but also subject to level effects.

The results from the model indicate that the cost of business cycle fluctuations is of two orders of magnitude higher than in the exogenous growth variant of the model. The presence of large welfare effects and the ability to potentially affect the growth rates and volatility of the economy through appropriate industrial policy creates space for policy intervention via static and countercyclical subsidies. Of those the most positive welfare effect is achieved through countercyclical subsidies to incumbents' operating cost, as it prevents excessive exits and encourages more R&D spending. Additionally, I find that accounting for frictions in the labor market results in welfare gains from static subsidies to incumbents' operating cost, a result at odds with the endogenous growth models that abstract from this friction.

The paper is based on the neo-Schumpeterian endogenous growth paradigm, pioneered by Grossman and Helpman (1991) and Aghion and Howitt (1992). Following the seminal contribution by Klette and Kortum (2004), there is a frugal literature on the relationship between endogenous growth and firm dynamics. This paper is close in spirit to work by Acemoglu et al. (2013) who study the consequences of subsidy schemes for R&D expenditures and growth, and related works include Akcigit and Kerr (2010) and Acemoglu and Cao (2015). The common assumption in those papers is that the incumbent firms innovate on their own products in a neo-Schumpeterian quality-ladder setup. I contribute to that literature by considering similar underlying mechanisms in a stochastic setup, and I am able to analyze the effect of countercyclical subsidies.

The paper also belongs to the growing body of the literature concerned with firm level heterogeneity and dynamics. Bartelsman and Doms (2000) provide a review of the early literature focused on documenting productivity differences and growth across firms and linking those phenomena to aggregate outcomes. Foster et al. (2001) emphasize the role of cyclical entry for aggregate productivity growth. The role of entry and exit channels for macroeconomic dynamics has been recognized and studied by Hopenhayn (1992), Devereux et al. (1996), Campbell (1998), Jaimovich and Floetotto (2008), Bilbiie et al. (2012), Chatterjee and Cooper (2014) and Lee and Mukoyama (2015), although none of those works incorporate the full set of firm dynamics considered here. Messer et al. (2016) show using regional US data that low entry rates in the 2007-2009 period contributed significantly to low employment and labor productivity growth Clementi and Palazzo (2016) study full firm dynamics over the business cycle, although their analysis focuses on the firm-level investment in physical capital, rather than innovation, which is the core mechanism of this paper.

The model also features frictional labor market, subject to the search and matching friction in the tradition of Diamond (1982) and Mortensen and Pissarides (1994). I follow an approach proposed by Gertler and Trigari (2009) that assumes convex hiring costs and is remarkably successful in replicating the labor market dynamics.

Moreover, I incorporate financial frictions modeled by assuming the working capital requirement as in Christiano et al. (2010) and having a reduced form of financial shocks in the form of the spread between the deposit and lending interest rates. There already exists literature that recognizes the impact of financial disturbances on macroeconomic variables[1] and firm dynamics, especially in the context of Great Recession. Severo and Estevao (2010) use industry-level panel data from Canada and US to show that increases in the cost of funds for firms have negative effects on TFP growth. Fernandez-Corugedo et al. (2011) build a DSGE model with multiple components of the working capital channel and find that even under flexible prices a disruption to the supply of credit has large and persistent effects on the real economy. Ates and Saffie (2014) build an small open economy entry-driven endogenous growth model to analyze the effects of a sudden stop using Chilean plant-level data. They find that although during financial shortage entrants are usually better, but they are fewer, generating permanent loss of output and significant welfare costs. Siemer (2014) finds that tight financial constraints during the Great Recession were responsible for both low employment growth and firm entry rates. Christiano et al. (2015) build a medium-scale DSGE model to quantify the importance of several shocks during the Great Recession and find that financial frictions were driving a significant part of the macroeconomic variables' behavior. This paper builds on this literature by considering the effects of financial friction within a model that simultaneously accounts for rich firm dynamics, entry and exit decisions, and endogenous growth.

The remainder of the paper is organized as follows. The next section describes the model, focusing on presenting the problem of incumbents and potential entrants, and describing the labor market and financial frictions. The third section provides a description of the data sources, parameter values and the estimation procedure, and discusses the cyclical properties of the model economy against the data. The fourth section applies the model to quantify the relative importance of the shocks during the Great Recession. It also analyzes in depth the welfare properties of the model economy and presents the macroeconomic and welfare effects of applying several subsidy schemes, both static and counterfactual. The last section concludes.

[1]Brunnermeier et al. (2012) provide an exhaustive review of the macroeconomic effects of financial frictions.

2 Model

The model presented here is based on Bielecki (2017a) and Bielecki (2017b). Compared to the previous work, the model introduces two key changes. First, as I am interested in the effect of financial shocks on the model macroeconomy, I introduce the wedge between the interest rate depositors (households) receive and the interest rate borrowers (establishments) pay. This allows me to capture the effects of a temporarily increased risk premium. Second, in line with Christiano et al. (2010), I incorporate the working capital channel which was found to be an important amplifier of financial shocks to the real economy.

2.1 Households

The mass of representative households is normalized to unity. Each of the households is composed of a large family of workers who differ with respect to their employment status and skill level. Nevertheless, due to within-family sharing, all individuals enjoy identical levels of consumption. The representative household maximizes the lifetime expected utility:

$$U_0 = \mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\theta}}{1-\theta} \tag{1}$$

where c_t denotes per capita consumption, β represents the discount factor and θ is the inverse of the elasticity of intertemporal substitution.

Following Acemoglu et al. (2013), I assume that workers belong to either of the two skill groups: unskilled workers of aggregate mass $(1-s)$ supply labor to the production sector, while skilled of aggregate mass s are hired as managers or to perform research and development activities. Furthermore, within a time period an individual worker may be employed and receive wage income, or unemployed and receive unemployment benefits. Here I abstract from the possibility that an individual is inactive in the labor market. The budget constraint of a representative household is given by:

$$c_t + d_{t+1} = (1-s)\left[w_t^u n_t^u + b_t^u\left(1-n_t^u\right)\right] + s\left[w_t^s n_t^s + b_t^s\left(1-n_t^s\right)\right] + \left(1+r_t^d\right)d_t + t_t$$

where d_{t+1} is the end-of-period t stock of deposits supplied to the financial sector which yields interest at deposit rate r_t^d, w_t^u and w_t^s represent wage income of employed unskilled and skilled workers, respectively, while n_t^u and n_t^s are the employment rates, and b_t^u and b_t^s denote unemployment benefits. Finally, t_t represents the sum of all dividend payments and lump-sum transfers net of taxes received by the households.

The intertemporal optimization by households yields the Euler equation:

$$c_t^{-\theta} = \beta \mathrm{E}_t\left[\left(1+r_{t+1}^d\right)c_{t+1}^{-\theta}\right] \tag{2}$$

and it is also convenient to define the stochastic discount factor of the households:

$$\Lambda_{t,t+1} = \mathrm{E}_t\left[\left(\frac{c_{t+1}}{c_t}\right)^{-\theta}\right] \tag{3}$$

By assuming that all firms are ultimately owned by households, I impose the condition that their managers discount future profit streams according to the households' valuation depending on the expected relative marginal utilities of consumption.

2.2 Final goods producers

The perfectly competitive final goods producers purchase differentiated intermediate goods varieties and transform them into final goods via the CES aggregator:

$$Y_t = \left[\int_0^{M_t} y_t(i)^{\frac{\sigma-1}{\sigma}} \, \mathrm{d}i \right]^{\frac{\sigma}{\sigma-1}}$$

where Y_t is the aggregate final goods output, M_t is the mass of active intermediate goods producing establishments, $y_t(i)$ is the quantity demanded from the i-th producer, and σ is the elasticity of substitution between the varieties.

Solving the profit maximization problem of the final goods producers yields the following Hicksian demand function for the i-th variety:

$$y_t(i) = Y_t p_t(i)^{-\sigma}$$

where $p_t(i)$ denotes the i-th variety's price relative to the aggregate price index, which can be constructed as follows:

$$P_t = \left[\int_0^{M_t} P_t(i)^{1-\sigma} \, \mathrm{d}i \right]^{\frac{1}{1-\sigma}}$$

and where $P_t(i)$ is the nominal price for the unit of the i-th intermediate good.

2.3 Intermediate goods producers

The monopolistically competitive intermediate goods first have to bear a fixed cost of operation f_t, which represents the costs of management and other non-production activities. Subsequently they can produce by employing capital and unskilled labor services according to the following Cobb-Douglas function:

$$y_t(i) = Z_t k_t^p(i)^{\alpha} \left[q_t(i) n_t^p(i) \right]^{1-\alpha}$$

where Z_t denotes the stochastic aggregate TFP parameter, $k_t^p(i)$ and $n_t^p(i)$ are the employed capital and unskilled labor, respectively, and $q_t(i)$ represents the idiosyncratic quality level of the i-th variety at time period t. Parameter α describes the elasticity of intermediate goods output with respect to capital.

The intermediate goods producers choose such combination of capital and labor that minimizes their costs. As in Christiano et al. (2010) I assume that each producer has to finance a constant fraction ζ of both the capital rental cost and wage bill in advance of production by borrowing necessary funds at the lending rate r_t^l[2]. The solution of the cost minimization problem results in the following expression for the marginal cost:

$$mc_t^p(i) = \frac{\left(1+\zeta r_t^l\right)}{Z_t}\left(\frac{r_t^k}{\alpha}\right)^{\alpha}\left(\frac{\tilde{w}_t^u/q_t(i)}{1-\alpha}\right)^{1-\alpha}$$

where r_t^k is the real rental rate on capital and $\tilde{w}_t^u$ denotes the unskilled wage paid to the employment agency. Note that the marginal costs differ across intermediate goods producers due to their differentiated quality levels.

Given that the producers can freely change their prices on the period-by-perod basis, the optimal pricing strategy is achieved by applying a constant markup over the marginal cost:

$$p_t(i) = \frac{\sigma}{\sigma-1} mc_t(i)$$

As in Melitz (2003), I construct the aggregate quality index Q_t which will summarize the aggregate situation in the intermediate goods sector as if it was populated by mass M_t of producers each with the same quality level. The index is constructed by applying the following formula:

$$Q_t = \left[\int_0^{\infty} q^{(1-\alpha)(\sigma-1)}\mu_t(q)\,\mathrm{d}q\right]^{\frac{1}{(1-\alpha)(\sigma-1)}}$$

where $\mu_t(q)$ denotes the period t distribution of the idiosyncratic quality levels.

It is very useful to describe the situation of an individual producer by comparing its quality level to the aggregate quality index and expressing it in relative terms:

$$\phi_t(i) = \left(\frac{q_t(i)}{Q_t}\right)^{(1-\alpha)(\sigma-1)}$$

It can then be shown that the operating profit of an intermediate goods producer can be expressed as:

$$\pi_t^o(i) = \frac{Y_t}{\sigma M_t}\phi_t(i) - f_t$$

Moreover, the aggregate final goods output is given as follows:

$$Y_t = M_t^{\frac{1}{\sigma-1}} Z_t \left(K_t^p\right)^{\alpha}\left(Q_t N_t^p\right)^{1-\alpha} \tag{4}$$

where K_t^p and N_t^p denote, respectively, the aggregate employment of capital and unskilled labor in the production sector while the presence of the active producers mass in the expression results from the love-for-variety phenomenon. Note that in the long run the only source of economic growth is the continued increase in aggregate quality level over time, and both capital stock and output will grow at the corresponding rate.

[2]In principle Christiano et al. (2010) allow for the degree of pre-financing to differ between the payments to capital and labor, but eventually they also assume that they are equal. I impose this assumption from the start and save on notation.

2.4 Incumbents

In the previous subsection I have discussed the static problem of the intermediate goods producer, where the quality level was given. This subsection describes the problem in the dynamic setting, where the incumbent producers can engage in research and development activities to have a chance at increasing their varieties' quality. The relative quality level of the i-th variety in period $t+1$ is decided by the following lottery:

$$\phi_{t+1}(i) = \begin{cases} \iota\phi_t(i)/\eta_t & \text{with probability } \chi_t(i) \\ \phi_t(i)/\eta_t & \text{with probability } 1-\chi_t(i) \end{cases}$$

where ι denotes the size of an innovative step and η_t is the transformed rate of growth of the aggregate quality index, which individual producers take as given:

$$\eta_t = \left(\frac{Q_{t+1}}{Q_t}\right)^{(1-\alpha)(\sigma-1)}$$

The innovative success probability $\chi_t(i)$ is chosen endogenously by each producer and is a function of engaged R&D resources. The particular form of the success probability function is based on Pakes and McGuire (1994) and Ericson and Pakes (1995):

$$\chi_t(i) = \frac{ax_t(i)}{1+ax_t(i)}$$

where a is a parameter that describes the efficacy of R&D input $x_t(i)$. The R&D process requires hiring both capital and skilled labor:

$$x_t(i) = \frac{k_t^x(i)^\alpha\left[Q_t n_t^x(i)\right]^{1-\alpha}}{Q_t\phi_t(i)}$$

where $k_t^x(i)$ and $n_t^x(i)$ denote the employed of capital and skilled labor.

The presence of aggregate and relative quality levels in the expression lends itself to a very intuitive interpretation. Aggregate quality level in the numerator multiplies with R&D laborers as they have access to a pool of common knowledge. However, over time it is harder to come up with new ideas unless more resources are committed to R&D activities, which is captured by aggregate quality level in the denominator. Finally, the presence of relative quality level in the denominator represents the catch-up and headwind effects, depending on establishments' position in the quality distribution.

In the absence of the last channel, establishments with higher quality product would have comparative advantage over their competitors and the success probability would be an increasing function of establishment size. This however is at odds with the empirically observed regularity known as Gibrat's law, according to which firm growth rates and firm size are uncorrelated. Empirical evidence on the evolution of firms shows that either the Gibrat's law cannot be rejected for large enough firms (see e.g. Hall (1987)) or that the larger firms have slower rates of growth (see e.g. Evans (1987), Dunne et al. (1989) or Rossi-Hansberg and Wright (2007)).

The solution of the cost minimization problem results in the following expression for the marginal cost of R&D activities:

$$mc_t^x(i) = \left(1+\zeta r_t^l\right) Q_t^\alpha \left(\frac{r_t^k}{\alpha}\right)^\alpha \left(\frac{\tilde{w}_t^s}{1-\alpha}\right)^{1-\alpha} \phi_t(i) \equiv \left(1+\zeta r_t^l\right) \bar{mc}_t^x \phi_t(i)$$

where $\tilde{w}_t^s$ denotes the skilled wage paid to the employment agency and $\bar{mc}_t^x$ denotes the skilled marginal cost component common to all establishments.

To simplify the setup of the intermediate goods producer, I assume that managerial activities require identical combination of capital and skilled labor as R&D activities. The fixed cost of operation can then be rewritten as follows:

$$f_t = \left(1+\zeta r_t^l\right) \bar{mc}_t^x f$$

Furthermore, to simplify notation I define the cost of skilled input relative to the current aggregate final goods output:

$$\omega_t = \bar{mc}_t^x / Y_t \tag{5}$$

The real profit of an incumbent can then be expressed as the following function, affine in terms of the relative quality level:

$$\pi_t(i) = Y_t \left[\left(\frac{1}{\sigma M_t} - \left(1+\zeta r_t^l\right)\frac{\omega_t}{a}\frac{\chi_t(i)}{1-\chi_t(i)}\right)\phi_t(i) - \left(1+\zeta r_t^l\right)\omega_t f\right]$$

Since all producers with the same relative quality levels will behave identically, from now on I drop the subscript i. The value of a producer with relative quality level ϕ_t can be expressed as follows:

$$V_t(\phi_t) = \max_{\chi_t \in [0,1]} \left\{\pi_t(\phi_t, \chi_t) + \max\left\{0, \mathrm{E}_t\left[\beta \Lambda_{t,t+1}(1-\delta_t) V_{t+1}(\phi_{t+1}|\phi_t, \chi_t)\right]\right\}\right\}$$

where δ_t is an endogenous probability that a producer will receive a death shock and the $\max\{0, \cdot\}$ operator allows the producers with low enough relative quality levels to voluntarily exit when the expected continuation value turns negative.

As the aggregate quality level trends upwards over time, causing other variables to trend as well, the above expression does not lend itself well to casting in the recursive form. I employ the following stationarization:

$$v_t(\phi_t) = \frac{V_t(\phi_t)}{Y_t}$$

where $v_t(\phi_t)$ denotes now the value of a producer relative to the current aggregate final goods output. The normalized value function is now stationary and can be stated as:

$$v_t(\phi_t) = \max_{\chi_t \in [0,1]} \left\{\frac{\pi_t(\phi_t, \chi_t)}{Y_t} + \max\left\{0, \mathrm{E}_t\left[\beta \Lambda_{t,t+1}(1-\delta_t)\left(\frac{Y_{t+1}}{Y_t}\right) v_{t+1}(\phi_{t+1}|\phi_t, \chi_t)\right]\right\}\right\}$$

As the model will be solved via the perturbation methods, I need to split the population of producers into two groups: those for which the max $\{0,\cdot\}$ operator is not binding, and those that exit.

As far as the first group is concerned, for large enough ϕ_t the probability that the particular producer shall exit in the foreseeable future is negligible and the operator can be safely omitted. Then it is trivial to show that since the value function is a weighted sum of current and future profit flows, all of which are affine in ϕ_t, then the value function is also affine in ϕ_t and the following functional form can be imposed:

$$A_t + B_t\phi_t = \max_{\chi_t\in[0,1]} \left\{ \begin{array}{c} \left(\frac{1}{\sigma M_t} - \left(1+\zeta r_t^l\right)\frac{\omega_t}{a}\frac{\chi_t}{1-\chi_t}\right)\phi_t - \left(1+\zeta r_t^l\right)\omega_t f \\ +\mathrm{E}_t\left[\beta\Lambda_{t,t+1}\left(1-\delta_t\right)\left(\frac{Y_{t+1}}{Y_t}\right)\left(A_{t+1}+B_{t+1}\frac{\chi_t(\iota-1)+1}{\eta_t}\phi_t\right)\right] \end{array} \right\} \tag{6}$$

where A_t and B_t are state-dependent coefficients that fluctuate over the business cycle.

The first order and envelope conditions of those producers can be then stated as follows:

$$0 = -\left(1+\zeta r_t^l\right)\frac{\omega_t}{a}\frac{1}{\left(1-\chi_t\right)^2} + \mathrm{E}_t\left[\beta\Lambda_{t,t+1}\left(1-\delta_t\right)\left(\frac{Y_{t+1}}{Y_t}\right)\left(B_{t+1}\frac{(\iota-1)}{\eta_t}\right)\right] \tag{7}$$

$$B_t = \left(\frac{1}{\sigma M_t} - \left(1+\zeta r_t^l\right)\frac{\omega_t}{a}\frac{\chi_t}{1-\chi_t}\right) + \mathrm{E}_t\left[\beta\Lambda_{t,t+1}\left(1-\delta_t\right)\left(\frac{Y_{t+1}}{Y_t}\right)B_{t+1}\frac{\chi_t\left(\iota-1\right)+1}{\eta_t}\right] \tag{8}$$

As the relative quality level drops out of the above optimality conditions, one can conclude that all producers with high enough relative quality level will choose the same success probability, and their size and growth rate will be uncorrelated, as postulated by the Gibrat's law.

The second group consists of the producers with negative continuation value and they will exit at the end of the current period and optimally choose not to engage in R&D activities at all. Their value function is then also affine in relative quality level and is equal to:

$$v_t\left(\phi_t\right) = Y_t\left[\frac{1}{\sigma M_t}\phi_t\left(i\right) - \left(1+\zeta r_t^l\right)\omega_t f\right] \tag{9}$$

At this stage the above division does not account for producers with intermediate quality levels. I then impose that all continuing producers have to choose the same success probability as their higher quality competitors. This results in the extension of Equations 6 and 9 until they intersect at the relative quality level at which a producer is indifferent between exiting and continuing conditional on choosing the common R&D intensity. This level is given implicitly by the following condition:

$$\left(1+\zeta r_t^l\right)\frac{\omega_t}{a}\frac{\chi_t}{1-\chi_t}\phi_t^* = \mathrm{E}_t\left[\beta\Lambda_{t,t+1}\left(1-\delta_t\right)\left(\frac{Y_{t+1}}{Y_t}\right)\left(A_{t+1}+B_{t+1}\frac{\chi_t\left(\iota-1\right)+1}{\eta_t}\phi_t^*\right)\right] \tag{10}$$

Furthermore, I assume that the relative quality levels follow the Pareto distribution with power parameter equal to one, an often made assumption in the literature dealing with firm size distribution[3]. This allows me to provide a closed form expression for the mass of the exiting producers:

$$M_t^x = M_t \left(1 - \chi_{t-1}\right) \left(1 - \frac{\phi_{t-1}^*}{\phi_t^* \eta_{t-1}}\right) \tag{11}$$

2.5 Entrants

The mass of potential entrants is assumed to be unbounded, although it will be pinned down by the equilibrium conditions. Similar to incumbents, they engage in R&D activities, although in this case the successful outcome of the innovation process results in entry, rather than an improvement over the existing product.

The entry attempt requires hiring capital and skilled labor, both for the purpose of performing R&D and managerial activities. The cost function mirrors the incumbents' case and the normalized value of entry can be stated as:

$$v_t^e = \max_{\chi_t^e \in [0,1]} \left\{ -\left(1 + \zeta^e r_t^l\right) \omega_t \left(f^e + \frac{1}{a^e} \frac{\chi_t^e}{1 - \chi_t^e}\right) + \chi_t^e \mathrm{E}_t \left[\beta \Lambda_{t,t+1} \left(\frac{Y_{t+1}}{Y_t}\right) v_{t+1} \left(\phi_{t+1}^e\right)\right] \right\}$$

where χ_t^e is the desired entry probability, ζ^e denotes the share of factor rental costs that has to be paid in advance and borrowed at lending rate, f^e is the fixed cost of operation of potential entrants, a^e is the efficacy of R&D inputs in the case of entrants, and ϕ_{t+1}^e represents the expected relative quality level determined upon successful entry.

The first order condition of the potential entrants is given as follows:

$$0 = -\left(1 + \zeta^e r_t^l\right) \frac{\omega_t}{a^e} \frac{1}{\left(1 - \chi_t^e\right)^2} + \mathrm{E}_t \left[\beta \Lambda_{t,t+1} \left(\frac{Y_{t+1}}{Y_t}\right) v_{t+1} \left(\phi_{t+1}^e\right)\right] \tag{12}$$

The unbounded mass of potential entrants implies that whenever the expected value of entry is positive, more candidates engage in the attempts, driving up the effective costs, and ensuring that the free entry condition holds:

$$v_t^e = 0 \tag{13}$$

Following the observations of Acemoglu and Cao (2015) and Garcia-Macia et al. (2016) I assume that entrants enjoy a degree of entry advantage. To account for that and rule out limit pricing in equilibrium, I assume that entrants draw their quality from appropriately upscaled quality distribution of incumbents. As a consequence, the expected relative quality level upon entry is given by:

$$E_t \left[\phi_{t+1}^e\right] = \frac{\sigma}{\sigma - 1}$$

By denoting with M_t^e the mass of successful entrants, one can pin down the mass of effective potential entrants, which is then given by M_t^e / χ_t^e. Entry is constrained by the

[3] Axtell (2001) provides empirical support for this assumption.

supply of skilled resources and is implicitly given by:

$$\frac{\left(K_t^s\right)^\alpha \left(Q_t N_t^s\right)^{1-\alpha}}{Q_t} = M_t f + \left(M_t - M_t^x\right)\left(\frac{1}{a}\frac{\chi_t}{1-\chi_t}\right) + \frac{M_t^e}{\chi_t^e}\left(f^e + \frac{1}{a^e}\frac{\chi_t^e}{1-\chi_t^e}\right) \tag{14}$$

where the left hand side equals the effective supply of skilled input which is (on the right hand side) split between operating cost of incumbents, R&D activities by continuers, and finally operating cost and R&D activities of potential entrants.

The rate of change in the aggregate quality index depends on the intensity of the R&D activities performed by the incumbents and the mass of new entrants relative to active establishments. By assuming Pareto distribution of quality levels it is possible to derive the exact closed form expression for the rate of growth of the aggregate quality index:

$$\eta_t = \left(1 - \chi_t + \chi_t \iota\right)\left(1 - \frac{M_t^e}{M_{t+1}} + \frac{M_t^e}{M_{t+1}}\frac{\sigma}{\sigma - 1}\right) \tag{15}$$

Finally, the endogenous probability of incumbent receiving a death shock depends on the exogenous, constant component, and the rate of entry of new establishments that potentially creatively destroy existing establishments. There are three conditions under which an active establishment exits, and I assume that at the end of each period the events follow a specific order. First, the incumbents with relative quality level below ϕ_t^* exit "voluntarily" as their varieties become obsolete. Second, incumbents receive exogenous exit shocks. Finally, a fraction of incumbents are leapfrogged by entrants and thus creatively destroyed. Therefore, the mass of active establishments in the next period is given by:

$$M_{t+1} = M_t - M_t^x - \delta^{exo}\left(M_t - M_t^x\right) + \left[1 - \left(1 - \delta^{exo}\right)\left(M_t - M_t^x\right)\right] M_t^e$$

where δ^{exo} is the exogenous exit shock probability and the mass of successful entrants M_t^e is multiplied by the probability that an entrant draws an "unoccupied" location. As by definition creative destruction replaces an incumbent with an entrant, it does not directly affect the mass of active establishments. The expression for active establishment mass can be also written as:

$$M_{t+1} = M_t - M_t^x - \delta_t\left(M_t - M_t^x\right) + M_t^e \tag{16}$$

Then by comparing the two formulations one gets the following expression for endogenous exit shock probability:

$$\delta_t = 1 - \left(1 - \delta^{exo}\right)\left(1 - M_t^e\right) \tag{17}$$

Intuitively, the probability of not receiving an exit shock is a product of the probabilities of not receiving an exogenous shock and not being creatively destroyed, as the two are independent from each other.

2.6 Labor markets

The labor markets are assumed to be subject to the search and matching mechanism and the staggered real wages friction introduced in Gertler and Trigari (2009) and Gertler et al.

(2008). Moreover, to retain tractability, I follow Christiano et al. (2011) and relegate the hiring and wage bargaining activities to employment agencies who then sell labor services to incumbents and entrants at a uniform wage. Thus, despite retaining the feature of a non-degenerate distribution of wages paid to workers over the business cycle all firms pay the same and their problem is simplified.

The unskilled and skilled markets are assumed to be separated and they do not directly influence each other. Therefore, I present the equations governing the behavior of a representative labor market only, as the mechanics of the skilled labor market is symmetrical, and I omit superscripts to ease the exposition.

The mass of unemployed workers is equal to:

$$u_t = 1 - n_t \tag{18}$$

The pool of unemployed is matched with the available vacancies according to the following matching function:

$$m_t = \sigma_m u_t^{\psi} v_t^{1-\psi} \tag{19}$$

where the parameter σ_m describes the efficiency of the matching process and ψ is the elasticity of matches with respect to the mass of unemployed.

The job finding probability p_t and job filling probability q_t can be obtained via the following transformation:

$$p_t = m_t / u_t \tag{20}$$

$$q_t = m_t / v_t \tag{21}$$

The employment agencies will choose the hiring rate defined as:

$$x_t = \frac{m_t}{n_t} \tag{22}$$

And the aggregate employment follows the law of motion:

$$n_{t+1} = (\rho + x_t)\, n_t \tag{23}$$

where $1 - \rho$ is the constant separation rate.

The individual employment agency supplies labor to producers at the common wage, but the wages paid to workers potentially differ between the agencies. Conditional on the offered wage, an employment agency chooses its hiring rate that maximizes the net value of an additional hired worker, subject to convex costs with respect to its hiring rate:

$$J_t(j) = \max_{x_t(j)} \left\{ \begin{array}{c} \tilde{w}_t - w_t(j) - \frac{\kappa}{2} x_t^2(j) \\ +(\rho + x_t(j))\, \mathrm{E}_t\left[\beta \Lambda_{t,t+1} J_{t+1}(j)\right] \end{array} \right\}$$

Solving the above problem results in the following first-order conditions:

$$\kappa x_t(j) = \mathrm{E}_t\left[\beta \Lambda_{t,t+1} J_{t+1}(j)\right]$$

$$\kappa x_t(j) = \mathrm{E}_t\left[\beta \Lambda_{t,t+1}\left[\tilde{w}_{t+1} - w_{t+1}(j) + \frac{\kappa}{2} x_{t+1}^2(j) + \rho \kappa x_{t+1}(j)\right]\right]$$

On the worker side, the values of being in the employed and unemployment states are given by the following formulas:

$$\mathcal{E}_t(j) = w_t(j) + \mathrm{E}_t\left[\Lambda_{t,t+1}\left[\rho\mathcal{E}_{t+1}(j) + (1-\rho)\,\mathcal{U}_{t+1}\right]\right]$$
$$\mathcal{U}_t = b_t + \mathrm{E}_t\left[\Lambda_{t,t+1}\left[p_t\mathcal{E}_{t+1} + (1-p_t)\,\mathcal{U}_{t+1}\right]\right]$$

where the unemployed worker engages in undirected search, resulting in the expected value of being newly hired expressed as:

$$\mathcal{E}_t \approx \int \mathcal{E}_t(w_t)\,\mathrm{d}G(w_t)$$

where G denotes the cumulative distribution of wages and the above approximation is valid to a first order.

Accordingly, the surplus of a worker employed by agency j and the average surplus of newly hired workers equal:

$$H_t(j) = \mathcal{E}_t(j) - \mathcal{U}_t$$
$$H_t = \mathcal{E}_t - \mathcal{U}_t$$

And the surplus of being a new hire can be expressed as:

$$H_t(j) = w_t(j) - b_t + \mathrm{E}_t\left[\beta\Lambda_{t,t+1}\left[\rho H_{t+1}(j) - p_t H_{t+1}\right]\right]$$

The wage bargaining follows the Nash bargaining procedure, although involved parties realize that the wage is not renegotiated on a period-by-period basis. Moreover, new hires receive the wage prevailing at the employment agency. Once the agency receives a signal to renegotiate, the wage maximizes the following Nash product:

$$w_t(r) = \arg\max H_t(r)^{\psi}\,J_t(r)^{1-\psi}$$

subject to the staggered wage contract friction:

$$w_t(j) = \begin{cases} w_t(r) & \text{with probability } 1-\lambda \\ w_{t-1}(j)\cdot Q_t/Q_{t-1} & \text{with probability } \lambda \end{cases}$$

where in the case of being unable to renegotiate the wages are updated according to the rate of growth of aggregate quality index. This assumption is similar to inflation indexation often assumed in the Calvo schemes, popularized by Christiano et al. (2005), and ensures that the wage dispersion is a second-order phenomenon and can be omitted under first-order approximation, facilitating solution. The Calvo friction implies that a wage contract lasts on average for $1/(1-\lambda)$ periods and the average wage evolves as follows:

$$w_t = \lambda\frac{Q_t}{Q_{t-1}}w_{t-1} + (1-\lambda)\,w_t(r) \tag{24}$$

I also already impose the Hosios (1990) condition that both sides' bargaining power correspond to matching function elasticities. The solution of the Nash bargaining problem results in the conventional surplus sharing formula:

$$\psi J_t(j) = (1-\psi) H_t(j)$$

While Gertler and Trigari (2009) consider a case where the above formula takes into account the horizon effect of the agency, the effect disappears under assumption that the wage bargaining and hiring decisions are simultaneous, i.e. internalizing the first order condition of the employment agency[4].

If the wages could be renegotiated each period, the contract wage (which I call flexible wage) would be identical across all employment agencies and equal to:

$$w_t^f = \psi\left(\tilde{w}_t + \frac{\kappa}{2}x_t^2 + p_t\kappa x_t\right) + (1-\psi) b_t \tag{25}$$

Under the case of staggered contracts the contract wage is given by:

$$\begin{aligned}\Delta_t w_t(r) = {} & w_t^f + \psi\left(\frac{\kappa}{2}\left(x_t^2(r) - x_t^2\right) + p_t\kappa\left(x_t(r) - x_t\right)\right) \\ & + (1-\psi) p_t \mathrm{E}_t\left[\beta\Lambda_{t,t+1}\lambda\Delta_{t+1}\left(w_{t+1} - w_{t+1}(r)\right)\right] \\ & + \rho\lambda \mathrm{E}_t\left[\beta\Lambda_{t,t+1}\Delta_{t+1}w_{t+1}(r)\right]\end{aligned} \tag{26}$$

where:

$$\Delta_t = \mathrm{E}_t \sum_{s=0}^{\infty} (\beta\rho\lambda)^s \Lambda_{t,t+s}\frac{Q_{t+s}}{Q_t}$$

The above equation emphasizes the presence of spillovers of economy-wide wages on the bargaining wage. Intuitively, more intensive hiring by an agency requires also higher bargained wages, which are also upwardly pressured by the future average wage.

2.7 Capital goods producers and the financial system

Perfectly competitive capital goods producers are also the owners of the capital stock which they rent to the establishments. They also borrow from the financial intermediary, at the lending interest rate r_t^l, in order to finance investment in new capital. Therefore, they aim to maximize the expected discounted flow their profits, expressed as:

$$\Pi_t^k = r_t^k K_t - I_t + L_{t+1}^k - \left(1 + r_t^l\right) L_t^k$$

where I_t is aggregate investment and L_t^k are loans from the financial intermediary. Physical capital accumulation is subject to the standard constraint:

$$K_{t+1} = I_t + (1-dp) K_t \tag{27}$$

[4]In any case, the quantitative impact of the horizon effect is negligible.

The solution of the capital goods producers' problem yields the following equality between the lending rate and the capital rental rate net of depreciation:

$$r_t^l = r_t^k - dp \tag{28}$$

The financial intermediaries collect deposits from households and lend them to two types of entities: intermediate goods producers and potential entrants, and capital producers. The profit of the intermediaries is given by:

$$\Pi_t^f = \left(1 + r_t^l\right) L_t^k + \left(1 + r_t^l\right) L_t^e - \left(1 + r_t^d\right) d_t + d_{t+1} - L_{t+1}^k - L_{t+1}^e$$

where L_t^e denote loans to establishments to finance their working capital requirement, and subject to the loanable funds constraint:

$$L_{t+1}^k + L_{t+1}^e \leq d_{t+1}$$

The financial intermediaries are owned by the households and discount the future in the same manner. Here I make the assumption that the financial intermediaries enjoy a degree of market power that drives a wedge between the deposit and lending interest rates, such that:

$$r_t^l = r_t^d + sp_t \tag{29}$$

where sp_t is the spread between the interest rates. While taken literally the variation in spread would imply that the market power of banks is changing over time, it can be also interpreted as a reduced-form way to capture the frictions in the financial markets. The spread is assumed to evolve according to the following AR(1) process:

$$\log sp_t = (1 - \rho_{sp}) \log sp_{ss} + \rho_{sp} \log sp_{t-1} + \varepsilon_{sp,t} \tag{30}$$

2.8 Market clearing

The markets for factors of production clear:

$$N_t^p = (1 - s)\, n_t^u \quad \text{and} \quad N_t^s = s n_t^s \tag{31}$$

$$K_t = K_t^p + K_t^s \tag{32}$$

As all profits are paid to the households, the budget constraint results in the standard resource constraint:

$$Y_t = C_t + I_t + \kappa^u \left(x_t^u\right)^2 N_t^p + \kappa^s \left(x_t^s\right)^2 N_t^s \tag{33}$$

Finally, the process for aggregate productivity is assumed to follow the standard AR(1) form:

$$\log Z_t = \rho_Z \log Z_{t-1} + \varepsilon_{Z,t} \tag{34}$$

3 Data and results

3.1 Data, calibration and estimation

The data used in the paper come from a couple of sources. The data on establishment dynamics comes from the US Bureau of Labor Statistics (BLS) Business Employment Dynamics (BDM) database. The BDM, based on the Quarterly Census on Employment and Wages (QCEW) records changes in the employment level of more than 98% of economic entities in the US, which also allows to track the cyclical behavior of establishments. Unfortunately, the data series is relatively short, starting as late as of 1992q3. The data on GDP and its components come from the US Bureau of Economic Analyses (BEA). The data on labor market statistics come predominately from the BLS. To construct the data on vacancies the data from the JOLTS survey, available from December 2000 were spliced with the Composite Help Wanted Index provided by Barnichon (2010). Following Christiano et al. (2014), the series for interest spread was chosen as the Moody's Seasoned BAA Corporate Bond Yield Relative to Yield on 10-Year Treasury Constant Maturity, provided by the Federal Reserve Bank of St. Louis, starting in April 1953. The data on R&D spending were taken from the BEA and the National Science Foundation (NSF). The NSF also provides the data on Full-Time Equivalent number of employees performing R&D, although the series ends in 2008.

The model is calibrated to replicate key features of the US economy. The parameters that influence the steady state of the economy are calibrated to reflect the long-run averages in the US data and are summarized in Table 1. The average quarterly spread was calculated directly from the corresponding data series. The degree of pre-financing was taken from Christiano et al. (2010). I decided not to differentiate the pre-financing parameters across incumbents and entrants. The values of parameters governing the behavior of the labor markets were taken from previous literature. Differentiated separation rates for unskilled and skilled workers are taken from Cairo and Cajner (2017) and adapted to the quarterly model setup. The adjustment cost parameters were chosen to match the average job finding probability in the US, which Shimer (2005) reports to be equal to 0.45 and Cairo and Cajner (2017) document that the job finding probabilities differ only slightly among the workers' education groups. As in Shimer (2005) the unemployment benefits are assumed to be equal to 40% of the steady state wage. Following Gertler and Trigari (2009) I set the elasticity of matches to unemployment to 0.5 and impose the Hosios (1990) condition that the bargaining power parameters correspond to matching elasticities. Finally, I set the matching efficiency parameter to match the observed average vacancy to unemployment ratio to 0.54.

Both the capital share of income and quarterly depreciation rate are set to values ubiquitous in the business cycle literature. Note that since in the model firms generate positive profits, the labor share of income is lower than 1 - capital share. The discount factor, which in the calibration process depends on the value of elasticity of intertemporal substitution, is chosen so that the average annual net deposit interest rate is equal to 4.75%, which together with the assumed average spread implies that the lending rate, equal to the rate of return on capital, equals 6.65%, a value consistent with literature, see e.g. Nishiyama and Smetters

(2007). The share of skilled workers is picked to be in the middle of the plausible range of values proposed by Acemoglu et al. (2013) and corresponds to the value used by Bielecki (2017a) and adjusted to account for the presence of unemployment in the model. Finally, the set of parameters governing the establishment dynamics is calibrated to match specific moments reported in Table 2. As I have 6 moments to match with 8 free parameters, I impose a constraint that the R&D efficiency parameter and fixed cost are equal for both incumbents and entrants.

Table 1: Calibrated parameters affecting the steady state

Parameter	Description	Value	Justification
sp_{ss}	Average quarterly spread	0.0047	Annualized spread = 1.9%
ζ, ζ^e	Working capital share	0.75	Christiano et al. (2010)
ρ^u	Unskilled retention rate	0.9725^3	Cairo and Cajner (2017)
ρ^s	Skilled retention rate	0.99^3	Cairo and Cajner (2017)
κ^u	Unskilled hiring cost	2	Unskilled job finding probability
κ^s	Skilled hiring cost	15.8	Skilled job finding probability
b^u	Unskilled unemp. benefit	0.14	40% of steady state unskilled wage
b^s	Skilled unemp. benefit	0.41	40% of steady state skilled wage
ψ	Elasticity of matches	0.5	Gertler and Trigari (2009)
σ_m	Matching efficiency	1.7	Average tightness = 0.54
α	Capital share of income	0.3	Standard
dp	Capital depreciation rate	0.025	Standard
β	Discount factor	0.9998	Annual net interest rate of 4.75%
s	Share of skilled workers	0.1039	Bielecki (2017a)
ι	Innovative step size	1.016	Annual pc. GDP growth
δ^{exo}	Exog. exit shock prob.	0.0174	Exit rate
a, a^e	R&D efficiency	7.96	Expansions = contractions
f, f^e	Operating fixed cost	0.94	Share of R&D in GDP
θ	Inverse of IES	2.3	Share of investment in GDP
σ	Elasticity of substitution	4.9	Share of R&D employment

Table 2: Long-run moments: comparison of model and data

Description	Model	Data	Source
Annual pc. GDP growth	2.06%	2.08%	BEA, 1948q1-2016q4
Exit rate	3.06%	3.07%	BDM, 1992q3-2016q4
Relative share of expanding estabs.	1.01	1.01	BDM, 1992q3-2016q2
Share of R&D in GDP	2.18%	2.23%	BEA, 1948q1-2016q4
Share of investment in GDP	17.50%	17.17%	BEA, 1948q1-2016q4
Share of R&D employment	1.31%	0.98%	NSF & CBP, 1964-2008

To obtain the values of parameters that do not affect the steady state but govern the cyclical behavior of the model, I employ the estimation procedure. The prior distributions were chosen to be relatively uninformative, and in particular the prior distribution for the renegotiation frequency parameter was set to uniform on the unit interval. Table 3 in the Appendix contains full information on the priors used.

The estimation makes use of two observable data series. The first one is the growth rate of Real Gross Domestic Product divided by the Labor Force, observed from 1948q2-2017q2. An advantage of the model with explicitly modeled long-run growth is that there is no need to detrend the data and valuable information is retained. The second is the demeaned spread between BAA and long-term government bonds. The model was estimated using standard Bayesian procedures with help of Dynare 4.5 and results were generated using two random walk Metropolis-Hastings chains with 200,000 draws each with an acceptance ratio of 0.24.

Table 3 presents the estimation results. The data were clearly informative about the estimated parameters, as the posterior and prior means differ and the highest posterior density (HPD) intervals are relatively tight. This observation can be also confirmed by comparing the plots of prior and posterior densities displayed in Figure 4.

The most interesting parameter is the one regulating the contract renegotiation probability, and its value implies that wage contracts last on average for 6 quarters. This value is slightly higher than assumed by Gertler and Trigari (2009) in their calibrated model, where they consider average duration of 9 and 12 months, and also higher than estimated by Gertler et al. (2008) where contracts last for 3.5 quarters[5]. However, assuming this value of the parameter yields excellent performance in case of labor market variables.

[5]Note however that Gertler et al. (2008) impose a relatively tight prior on this parameter.

Table 3: Prior and posterior means of parameters affecting cyclical behavior

Parameter	Description	Prior mean	Post. mean	90% HPD interval
λ	Calvo wage contract prob.	0.5	0.858	$[0.766, 0.950]$
ρ_Z	Autocorr. of prod. process	0.7	0.939	$[0.896, 0.991]$
σ_Z	Std. dev. of prod. shock	0.01	0.012	$[0.011, 0.013]$
ρ_{sp}	Autocorr. of spread process	0.7	0.930	$[0.895, 0.968]$
σ_{sp}	Std. dev. of spread shock	0.1	0.163	$[0.151, 0.174]$

3.2 Model performance and impulse response functions

The data moments were generated on the sample 1948q1-2016q4, with the exception of vacancies and tightness, available from 1951q1, and establishment dynamics, available from 1992q3. The variables trending with population growth, such as GDP and number of establishments, were divided by the Civilian Labor Force and subsequently detrended with Hodrick-Prescott filter.

Table 4 presents the comparison of the Hodrick-Prescott filtered moments between the model and data. The upper section of the table is concerned with output and its components, as well as R&D expenditures. The model fits the data very well for output and its components, and only fails to account for much weaker correlation of R&D expenditures with output.

The middle section of the table focuses on variables pertaining to the operations of the labor market. The model wages are stronger correlated with output and have higher autocorrelation than in the data, and model hours are not as volatile as in the data. However, the model is very successful in matching the cyclical behavior of unemployment, vacancies and tightness, achieving nearly perfect fit.

The final section presents the moments related to the establishment dynamics. Although the fit is a bit worse than in the case of previously discussed variables, most of the model moments remain close to their data counterparts, with the exception that the model predicts much smaller volatility of establishment dynamics. The model also predicts that the establishment mass is slightly negatively correlated with output, even though the correlation of net entry with output is almost exactly the same as in the data. A brief look at the impulse response functions in Figure 1 reveals that this result is most likely driven by a small and short-lived decrease in the mass of establishments immediately after the shock hits, and for the subsequent periods the mass of active establishments moves in tandem with output. In the case of interest rate spread shocks, Figure 2 shows that output and establishment mass comove.

To sum up, although the model is not able to match the data perfectly, the fit is more than satisfactory and provides a solid foundation for further analysis.

Table 4: Business cycle moments: comparison of model and data

	Standard deviation		Correlation with Y		Autocorrelation	
Variable	Data	Model	Data	Model	Data	Model
Output	1.58	1.58	1.00	1.00	0.82	0.83
Consumption	0.87	0.69	0.78	0.90	0.82	0.75
Investment	4.54	5.33	0.76	0.97	0.87	0.89
R&D	2.36	2.26	0.32	0.91	0.89	0.92
Wages	0.95	0.73	0.10	0.41	0.68	0.96
Hours	1.36	0.76	0.86	0.92	0.89	0.89
Unemployment	12.76	11.96	-0.77	-0.82	0.89	0.89
Vacancies	13.78	13.47	0.83	0.92	0.91	0.90
Tightness	26.00	24.45	0.82	0.89	0.91	0.91
Establishments	0.62	0.31	0.71	-0.38	0.87	0.89
Expansions	2.84	0.58	0.82	0.78	0.75	0.89
Contractions	2.38	0.76	-0.11	-0.93	0.69	0.89
Net Entry	0.31	0.15	0.31	0.18	0.24	0.45

Figure 1 displays the impulse response functions to a standard deviation productivity shock. An increase in productivity raises output both directly and indirectly, through higher investment and hiring rates, which in turn cause an increase in physical capital stock and hours worked. The response of output to the shock is highly persistent, both due to labor market frictions and the endogenous quality component which permanently shifts output upwards. Expenditures on R&D are also procyclical and persistent.

Staggered wage contract friction prevents wages from responding strongly on impact of the shock, as a large fraction of employment agencies is not allowed to renegotiate wages. Over time, the wage renegotiations take place, and the response of wages exhibits a hump-shaped pattern, reaching the peak at around 3 years after the initial shock. Increased productivity of labor induces the employment agencies to post vacancies, increasing labor market tightness.

Positive productivity shock incentivizes incumbents to increase their R&D expenditures and consequently success probability, and as a consequence the mass of expanding establishments increases, and the mass of contracting establishments decreases. At impact elevated incumbents' demand for scarce resources, especially for skilled labor, results in a temporary decline in net entry rates. However, as more skilled employees become available, net entry turns positive and translates to an increase in the active establishments mass. The rate of growth of aggregate quality index is higher than along the balanced growth path, due to both higher R&D intensity and entry rates. This faster pace of growth is at first maintained by both higher skilled employment and bigger capital stock, although after around 4 years employment returns to its balanced growth path level, leading to a decrease in the rate of quality growth. Nevertheless, more abundant physical capital allows the economy to continue growing faster, and eventually the level of quality relative to balanced growth path trend flattens out and stabilizes at a level around 7% higher than it would be if the shock never happened.

Figure 2 displays the impulse response functions to a standard deviation interest spread shock. Broadly speaking, an increase in the wedge between the deposit and lending rates generates effects opposite to the positive productivity shock, and their quantitative size is of order of magnitude smaller. Immediately on impact investment decreases, as it is now more costly to produce new units of physical capital, and consumption rises in response to lower deposit interest rates. Expenditures on R&D initially increase, as incumbents face lower risk of being creatively destroyed due to decreased entry, but after about a year drop below the balanced growth path trend as the recession deepens. Both hours worked and wages decrease in a hump-shaped pattern, while unemployment increases. Creating new vacancies is discouraged, and as a result the labor market becomes less tight. Increased costs of lending deter entry which remains depressed for about 5 years after the initial shock, which also causes a decrease in the mass of establishments. Aggregate quality level remains near its trend level for around 2 years following the shock, as expansions and net entry move in opposite directions. After that period both depressed entry and incumbents' R&D intensity translate to the downward deviation of the quality level from trend, which eventually is lowered by about 0.85% relative to its trend path. Thus the financial shocks, compared to productivity shocks, create similar, although smaller in magnitude, shifts in the balanced growth path of the economy. As a robustness check, Figures 5 and 6 in the Appendix presents the Bayesian impulse response functions taking into account parameter uncertainty. All of the results remain unchanged.

Figure 1: Impulse response functions to standard deviation productivity shock

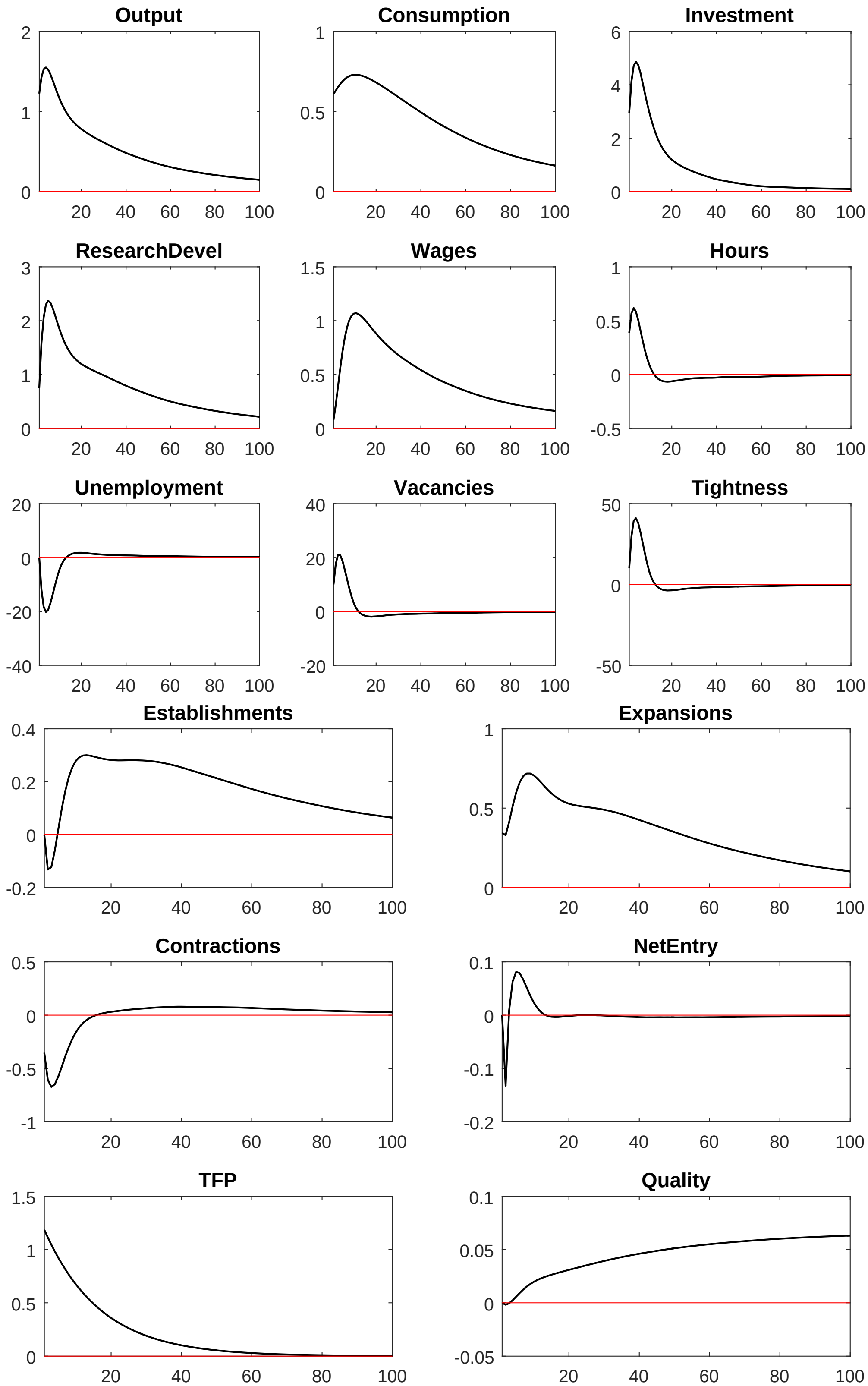
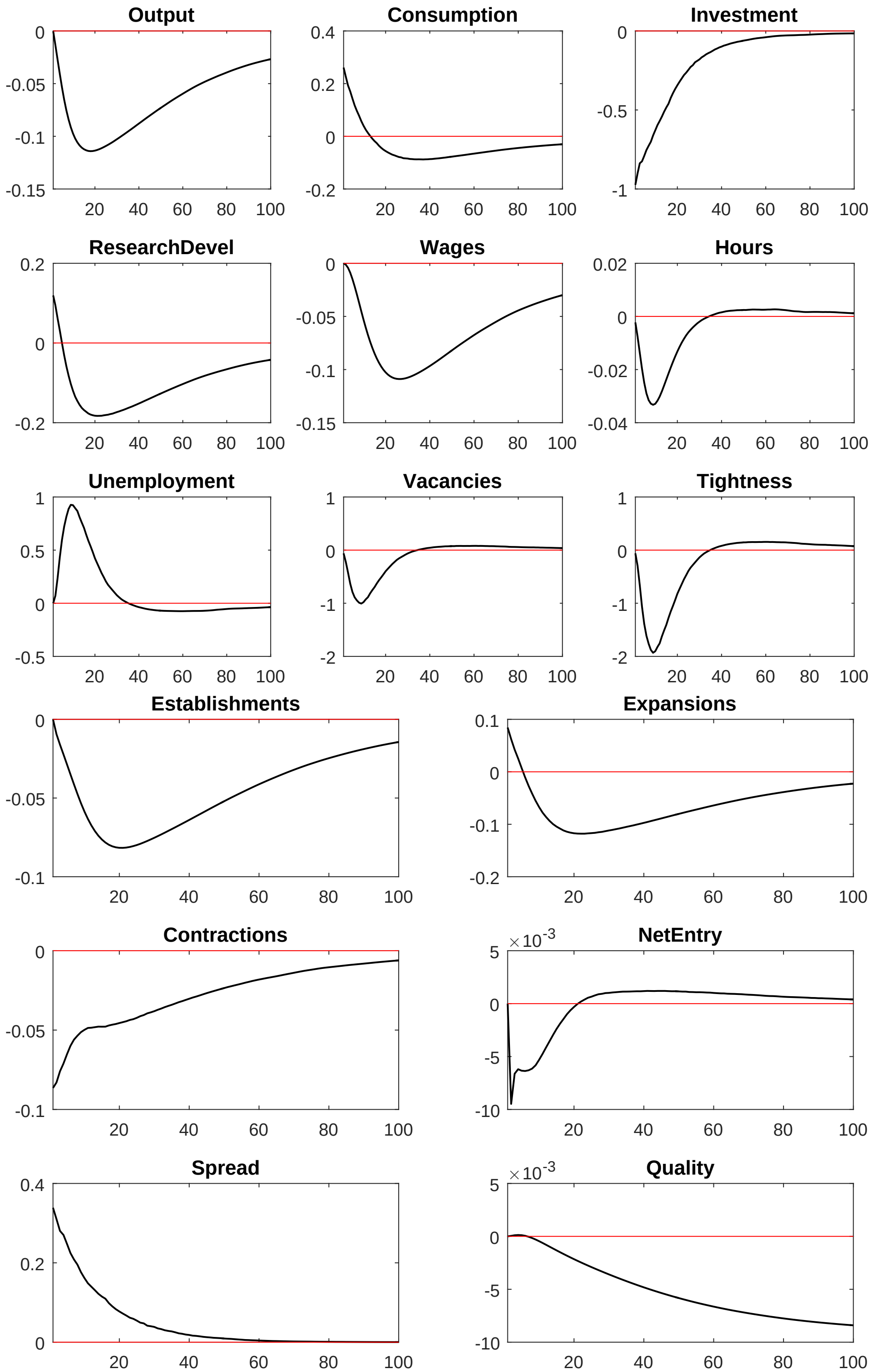


Figure 2: Impulse response functions to standard deviation interest rate spread shock



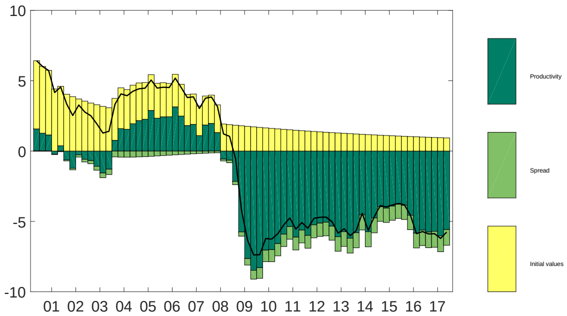
4 Long shadows of financial shocks

4.1 The experience of Great Recession

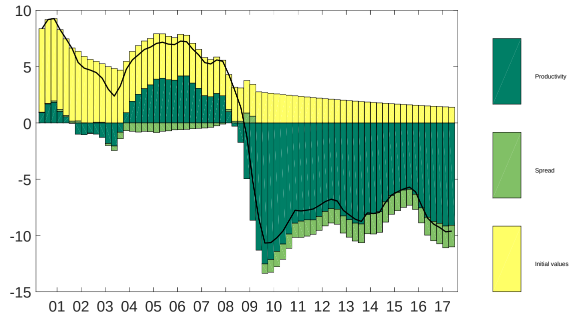
The model features mechanisms through which temporary shocks translate to permanent shifts to the balanced growth path of the economy. Therefore it is an attractive laboratory to study the experience of the Great Recession.

The Great Recession has been associated with the largest output drop in the postwar economic history of the United States, which until now remains around 10% below its pre-recession trend. A similar behavior was observed for the R&D expenditures, although the drop was even deeper than for output. Increased establishment exits and depressed entry has resulted in fewer active establishments.

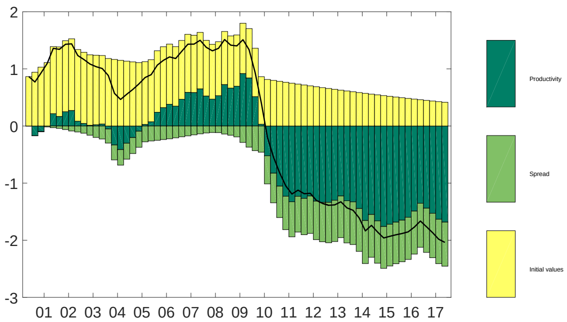
Figure 4.1 presents the shock decomposition of key macroeconomic variables since the first quarter of 2000 until the second quarter of 2017. The financial shocks, modeled as increases in the spread between deposit and lending interest rates, account for a nontrivial fraction of the deviation of the variables from their trend. In particular, about a third of the total decline in the establishment mass is attributed to increased spreads, as they are especially harmful to entrants. Depressed entry rates and R&D expenditures result in continuing fall of the aggregate quality index. It has profound implications as while physical capital stock and employment levels can in principle return to their balanced growth path levels, a decline in the aggregate quality is of a more permanent nature and essentially pushes the economy to a balanced growth path below the pre-crisis one.



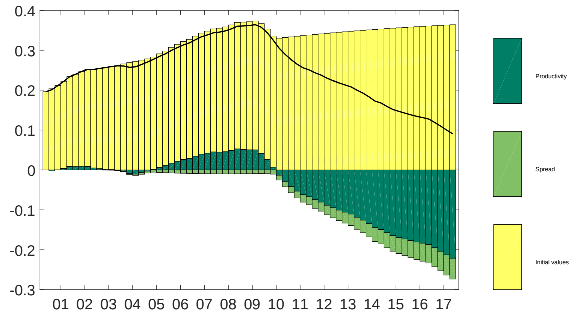
(a) Output



(b) Research & Development



(c) Establishments



(d) Quality Index

Figure 3: Shock decomposition: key macroeconomic variables since 2000

4.2 Policy implications

Since temporary shocks can exert level effects on the balanced growth path of the economy, this implies that business cycle fluctuations are associated with additional welfare costs compared to the models where growth results from exogenous processes.

The consumption equivalent method allows to quantify the welfare differences across states of world. The equivalent is equal to the lifetime percentage adjustment in the path of households' consumption that make them indifferent between "living" in two worlds. For the utility function assumed in this paper the equivalent-adjusted welfare is given by:

$$W_0\left(eq\right) = \mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{\left(\left(1+eq\right)c_t\right)^{1-\theta}}{1-\theta} = \left(1+eq\right)^{1-\theta} \mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\theta}}{1-\theta}$$

The consumption equivalent across two different worlds can be then computed as follows:

$$eq_{a,b} = \left(\frac{U_0^b}{U_0^a}\right)^{\frac{1}{1-\theta}} - 1$$

where U_0^a and U_0^b denote expected lifetime utilities in worlds a and b, respectively. Then $eq_{a,b}$ has the interpretation of which proportion of consumption the agent living in world a would we willing to forfeit in order to "move" to world b.

Table 5 presents the comparison of expected lifetime utilities in five distinct worlds. In the non-stochastic world the economy is not subject to shocks and always remains on its balanced growth path. In the two stochastic worlds the economy is affected by shocks but in the first of them growth is exogenous and the aggregate quality index increases at a constant rate. As a consequence, any welfare losses result from the volatility around the trend and are estimated to be quite low, in accordance with existing literature. The second stochastic world is identical to the model economy. Here welfare losses are significantly larger and stem from the fact that both shocks result in the level shifts of the consumption paths. Finally, the lower section of Table 5 is concerned with the relative importance of two shocks for welfare. It turns out that the spread shocks account for about a third of the total welfare costs.

Table 5: Welfare cost of business cycles

State of the world	Welfare	Consumption equivalent
Nonstochastic (BGP)	-172.84	–
Stochastic with exogenous growth	-172.97	0.05%
Stochastic with endogenous growth	-191.13	8.04%
Endogenous growth without spread shocks	-178.85	2.66%
Endogenous growth without prod. shocks	-185.52	5.60%

The presence of significant welfare costs of business cycles poses questions on whether economic policy can alleviate some of them. To answer them I examine the macroeconomic and welfare effects of applying several subsidy schemes. Those schemes fall into two groups: static and countercyclical subsidies. All subsidy schemes are financed via lump-sum taxation.

Static subsidies are designed to act as if a parameter in question was changed by 10%. The direction of change is always in the direction favored by the subsidized agent, e.g. a lowering of operation costs or increasing the R&D efficiency. Table 6 documents the results of applying. The first result column displays the average growth rate in stochastic equilibrium. The next two present the extent of change in the aggregate quality index in response to a standard deviation productivity shock, over the horizon of 20 and 100 quarters, respectively, while the following two columns do the same for the spread shock. Next column reports the expected lifetime utility measure, and the following the average unemployment rate in stochastic equilibrium. For ease of interpretation, the last column presents the opposite number to the consumption equivalent, so that the positive value of the statistic indicates welfare gain. As a rule of thumb, ceteris paribus households prefer if the aggregate growth rate is higher, volatility (understood as the extent of the reaction of aggregate quality in response to the shock) is lower and unemployment rate is lower.

In agreement with the endogenous growth literature I find that subsidizing R&D expenditures of incumbent establishments is strongly welfare improving, as both the average growth rate is increased and volatility is decreased, at a cost of slightly higher unemployment rate. Welfare gains are also associated with lowering the barriers to entry, either through lowering the fixed costs of prospective entrants or subsidizing their R&D activities. Contrary to the previous literature, e.g. Acemoglu et al. (2013), I find that subsidizing incumbents' operating costs is welfare improving. This discrepancy stems from the fact that although the subsidized economy exhibits lower rate of growth and higher volatility, those effects are dwarfed by gains from decreased churning in the labor market, the full extent of which become apparent only in the stochastic setting.

The lower section of the table presents the effects of subsidies that aim to reduce frictions in the financial markets. Subsidizing the working capital costs of incumbents lowers slightly the volatility of the economy and generates a small welfare gain, while subsidies to working capital of entrants do not have a significant welfare effect. Finally, subsidizing all borrowers in a manner that acts as if the average spread was lower decreases both volatility and average unemployment rate, resulting in welfare gain.

Table 6: Effects of static subsidies

	γ	ΔQ^Z_{20}	ΔQ^Z_{100}	ΔQ^{sp}_{20}	ΔQ^{sp}_{100}	U	u	-eq
Baseline	2.09	3.17	6.32	0.21	0.82	-191.13	5.65	–
f	2.06	3.33	6.63	0.24	0.87	-187.07	5.53	1.64%
f^e	2.09	3.18	6.36	0.22	0.82	-190.91	5.65	0.09%
a	2.15	3.08	6.12	0.22	0.80	-186.97	5.67	1.68%
a^e	2.10	3.17	6.34	0.22	0.82	-191.05	5.65	0.03%
ζ	2.10	3.16	6.32	0.21	0.82	-190.92	5.65	0.09%
ζ^e	2.10	3.17	6.32	0.21	0.82	-191.15	5.65	-0.01%
sp_{ss}	2.10	3.16	6.31	0.19	0.74	-189.28	5.63	0.75%

Table 7 presents the welfare effects of countercyclical subsidies. As they by construction do not impact significantly the economy's average growth rate or unemployment rate, those variables are not displayed. I consider two variants of subsidies: in the first, if output is observed at level 1% lower than trend, the subsidy increases by 0.5%. Conversely, in the times of boom the subsidy becomes a tax. In the second variant subsidy increases by 5% if the spread is 1 percentage point higher than on average.

The qualitative effects of the two subsidy variants are almost identical, and thus I will discuss only the effects of subsidies reacting to output. Intuitively, subsidy schemes that lower the volatility bring welfare gains. The biggest welfare gains are associated with subsidizing operating costs of active establishments, which gives support for policies aimed at supporting existing firms during recessions. On the other hand, countercyclical subsidies to incumbents' R&D activities are welfare deteriorating, as by redirecting limited resources towards incumbents it exacerbates the difficulties entrants face during downturns. Finally, subsidies to entrants carry small positive welfare gains, while subsidies to working capital have almost no impact on volatility and welfare.

Table 7: Effects of countercyclical subsidies

0.5% subsidy if output is 1% below trend

	ΔQ^Z_{20}	ΔQ^Z_{100}	ΔQ^{sp}_{20}	ΔQ^{sp}_{100}	U	-eq
Baseline	3.17	6.32	0.21	0.82	-191.13	–
f	3.11	6.16	0.21	0.79	-187.00	1.67%
f^e	3.16	6.31	0.21	0.82	-190.99	0.06%
a	3.30	6.60	0.22	0.86	-195.82	-1.88%
a^e	3.17	6.32	0.21	0.82	-191.08	0.02%
ζ	3.17	6.32	0.21	0.82	-191.17	-0.01%
ζ^e	3.17	6.32	0.21	0.82	-191.13	0.00%

5% subsidy if spread is 1 percentage point above average

	ΔQ^Z_{20}	ΔQ^Z_{100}	ΔQ^{sp}_{20}	ΔQ^{sp}_{100}	U	-eq
Baseline	3.17	6.32	0.21	0.82	-191.13	–
f	3.17	6.32	0.20	0.79	-189.28	0.75%
f^e	3.17	6.32	0.21	0.82	-191.07	0.02%
a	3.17	6.32	0.26	0.88	-193.20	-0.83%
a^e	3.17	6.32	0.21	0.82	-191.11	0.01%
ζ	3.17	6.32	0.21	0.82	-191.15	-0.00%
ζ^e	3.17	6.32	0.21	0.82	-191.13	0.00%

5 Conclusions

The Great Recession has resulted in a seemingly permanent level shift in many macroeconomic variables. This paper has presented an endogenous growth model where monopolistically competitive, heterogeneous establishments choose the level of R&D expenditures. The model economy is also subject to the search and matching friction in the labor market, as well as financial friction modeled as a reduced-form shock to the spread between deposit and lending interest rates. This setup generates volatile and procyclical R&D expenditure patterns and is consistent with the business cycle dynamics of GDP and its components, labor market variables, as well as establishment dynamics.

I find that both productivity and financial shocks affect the endogenous growth rate of the economy, resulting in level shifts in the balanced growth path. This significantly increases the estimate of the welfare costs of business cycles. As a consequence, economic policy can play an important role in alleviating the consequences of those shocks. I analyze the macroeconomic and welfare effects of a series of static and countercyclical subsidy schemes.

Regarding the static subsidies, I find that subsidizing R&D expenditures, as well as lowering barriers to entry, is welfare improving, in line with endogenous growth literature. At odds with this literature, static subsidies to incumbents' operating costs are also found to be welfare improving. This result stems from taking into account the effects of business cycle fluctuations in an economy with frictional labor and financial markets.

Regarding the countercyclical subsidies, I find that subsidizing R&D expenditures of active establishments is welfare deteriorating, as it redirects precious resources from more efficient uses. On the other hand, subsidizing incumbents' operating costs is welfare enhancing, regardless of whether the economy is hit by productivity or financial shock. This result supports implementing policies that aim to reduce exits of active establishments during recessions.

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A Appendix

A.1 Additional tables and figures

Table 8: Prior distributions of parameters

Parameter	Description	Distribution shape	Mean	Std. dev.
λ	Calvo wage contract prob.	Uniform $[0, 1]$	0.5	0.289
ρ_Z	Autocorr. of prod. process	Beta	0.7	0.175
σ_Z	Std. dev. of prod. shock	Inverse Gamma	0.01	∞
ρ_{sp}	Autocorr. of spread process	Beta	0.7	0.175
σ_{sp}	Std. dev. of spread shock	Inverse Gamma	0.1	∞

Figure 4: Prior and posterior distributions of estimated parameters

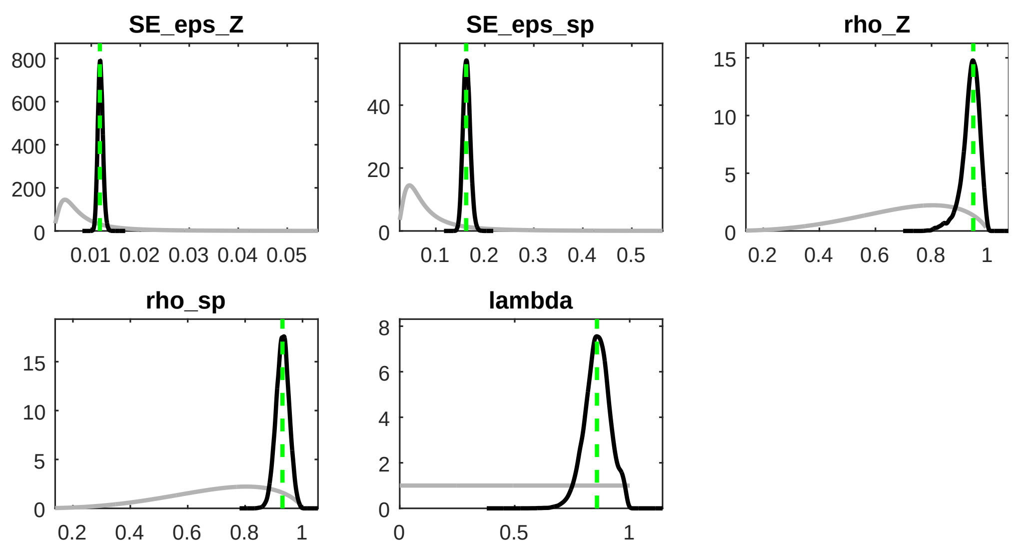


Figure 5: Bayesian impulse response functions to productivity shock

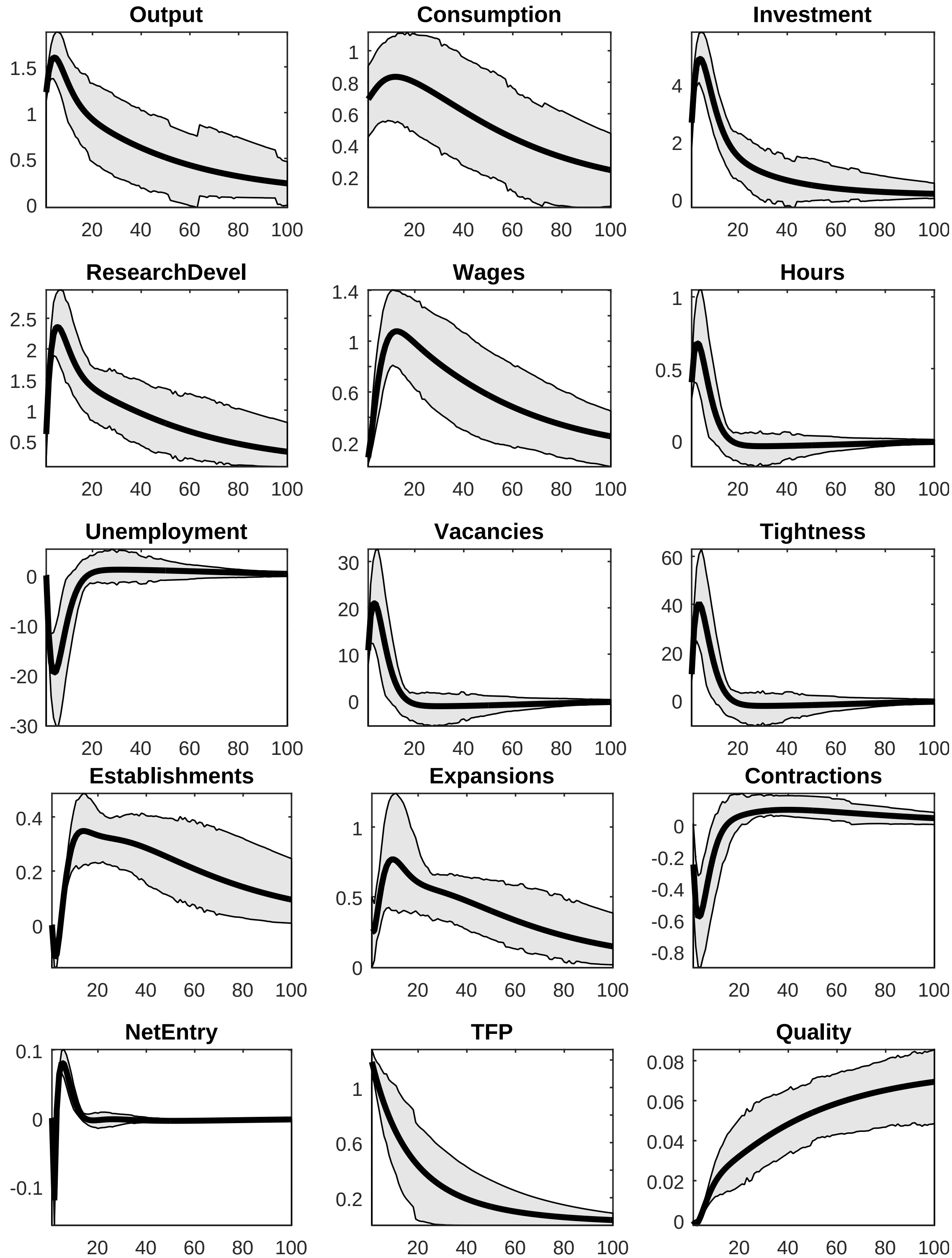
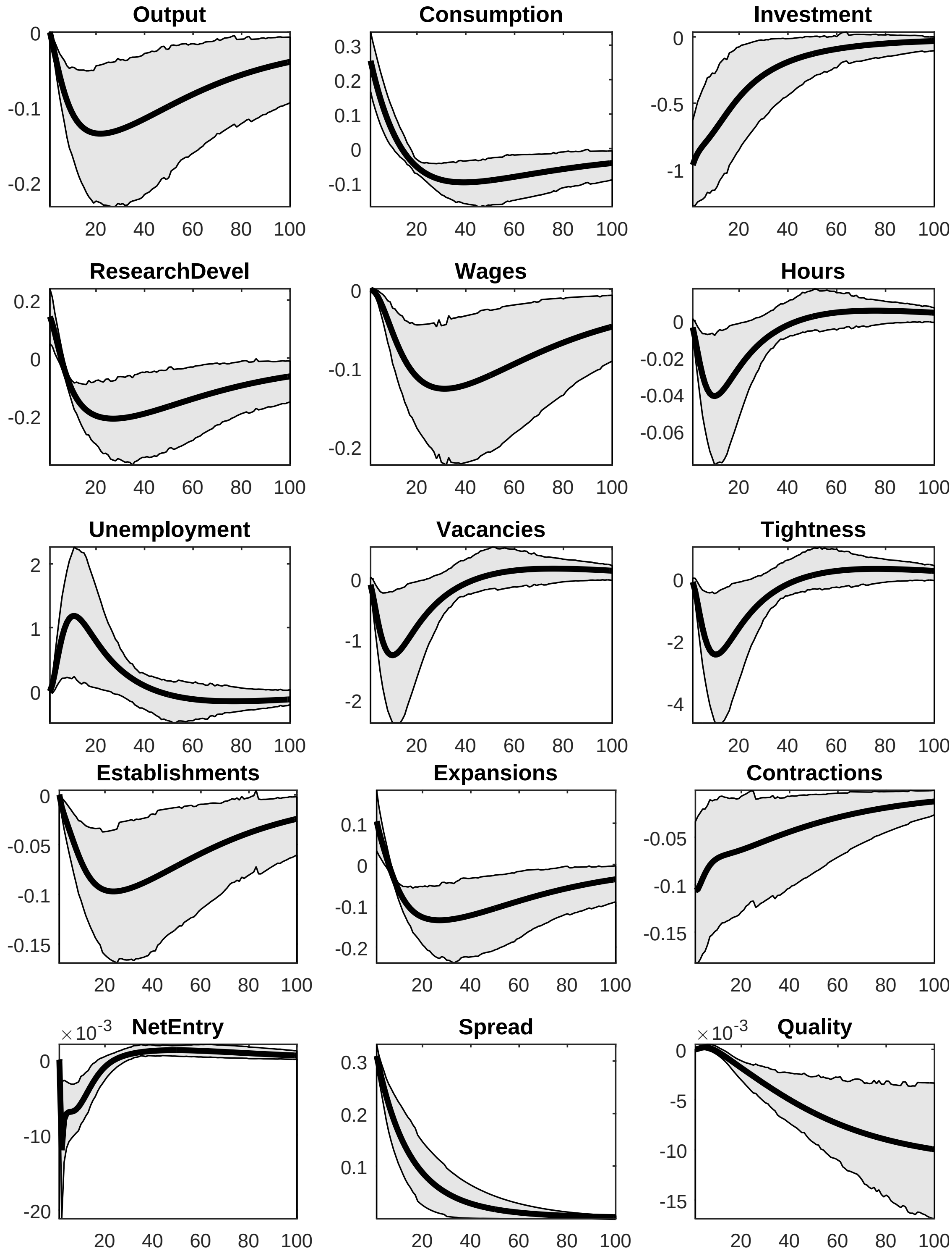


Figure 6: Bayesian impulse response functions to interest rate spread shock



A.2 Full set of stationarized model equations

Stationarized variables notation

$$\hat{X}_t \equiv X_t/Q_t$$

Stationarizing variables

$$g_t^Q \equiv Q_{t+1}/Q_t = \eta_t^{\overline{\frac{1}{(1-\alpha)(\sigma-1)}}} \tag{A.1}$$

$$\gamma_{t,t+1} \equiv Y_{t+1}/Y_t = g_t^Q \cdot \hat{Y}_{t+1}/\hat{Y}_t \tag{A.2}$$

Incumbents' problem

$$\phi_t = 1 \tag{A.3}$$

$$v_t = A_t + B_t\phi_t \tag{A.4}$$

$$\pi_t = \left(\frac{1}{\sigma M_t} - \left(1+\zeta r_t^l\right)\frac{\omega_t}{a}\frac{\chi_t}{1-\chi_t}\right)\phi_t - \left(1+\zeta r_t^l\right)\omega_t f \tag{A.5}$$

$$A_t + B_t\phi_t = \pi_t + \mathrm{E_t}\left[\Lambda_{t,t+1}\left(1-\delta_t\right)\gamma_{t,t+1}\left(A_{t+1} + B_{t+1}\frac{\chi_t\left(\iota-1\right)+1}{\eta_t}\phi_t\right)\right] \tag{A.6}$$

$$0 = -\left(1+\zeta r_t^l\right)\frac{\omega_t}{a}\frac{1}{\left(1-\chi_t\right)^2} + \mathrm{E_t}\left[\beta\Lambda_{t,t+1}\left(1-\delta_t\right)\gamma_{t,t+1}B_{t+1}\frac{\left(\iota-1\right)\phi_t}{\eta_t}\right] \tag{A.7}$$

$$B_t = \left(\frac{1}{\sigma M_t} - \left(1+\zeta r_t^l\right)\frac{\omega_t}{a}\frac{\chi_t}{1-\chi_t}\right) + \mathrm{E_t}\left[\beta\Lambda_{t,t+1}\left(1-\delta_t\right)\gamma_{t,t+1}B_{t+1}\frac{\chi_t\left(\iota-1\right)+1}{\eta_t}\right] \tag{A.8}$$

Entrants' problem

$$v_t^e = -\left(1+\zeta^e r_t^l\right)\omega_t\left(f^e + \frac{1}{a^e}\frac{\chi_t^e}{1-\chi_t^e}\right) + \chi_t^e\mathrm{E_t}\left[\beta\Lambda_{t,t+1}\gamma_{t,t+1}\left(A_{t+1} + B_{t+1}\frac{\sigma}{\sigma-1}\phi_{t+1}\right)\right] \tag{A.9}$$

$$0 = -\left(1+\zeta^e r_t^l\right)\frac{\omega_t}{a^e}\frac{1}{\left(1-\chi_t^e\right)^2} + \mathrm{E_t}\left[\beta\Lambda_{t,t+1}\gamma_{t,t+1}\left(A_{t+1} + B_{t+1}\frac{\sigma}{\sigma-1}\phi_{t+1}\right)\right] \tag{A.10}$$

$$v_t^e = 0 \tag{A.11}$$

Establishment dynamics

$$\delta_t = 1 - \left(1-\delta^{exo}\right)\left(1-M_t^e\right) \tag{A.12}$$

$$\left(1+\zeta r_t^l\right)\frac{\omega_t}{a}\frac{\chi_t}{1-\chi_t}\phi_t^* = \mathrm{E}_t\left[\beta\Lambda_{t,t+1}\left(1-\delta_t\right)\gamma_{t,t+1}\left(A_{t+1} + B_{t+1}\frac{\chi_t\left(\iota-1\right)+1}{\eta_t}\phi_t^*\right)\right] \tag{A.13}$$

$$M_t^x = M_t\left(1-\chi_{t-1}\right)\left(1 - \frac{\phi_{t-1}^*}{\phi_t^*\eta_{t-1}}\right) \tag{A.14}$$

$$M_{t+1} = \left(1-\delta_t\right)\left(M_t - M_t^x\right) + M_t^e \tag{A.15}$$

$$\eta_t = \left(1-\chi_t+\chi_t\iota\right)\left(1 - \frac{M_t^e}{M_{t+1}} + \frac{M_t^e}{M_{t+1}}\frac{\sigma}{\sigma-1}\right) \tag{A.16}$$

Skilled sector

$$\omega_t \hat{Y}_t = \left(\frac{r_t^k}{\alpha}\right)^{\alpha} \left(\frac{\hat{\tilde{w}}_t^s}{1-\alpha}\right)^{1-\alpha} \tag{A.17}$$

$$\left(\hat{K}_t^s\right)^{\alpha} (N_t^s)^{1-\alpha} = M_t f + (M_t - M_t^x)\left(\frac{1}{a}\frac{\chi_t}{1-\chi_t}\right) + \frac{M_t^e}{\chi_t^e}\left(f^e + \frac{1}{a^e}\frac{\chi_t^e}{1-\chi_t^e}\right) \tag{A.18}$$

$$\frac{r_t^k}{\hat{\tilde{w}}_t^s} = \frac{\alpha}{1-\alpha}\frac{N_t^s}{\hat{K}_t^s} \tag{A.19}$$

Unskilled sector

$$\hat{Y}_t = Z_t M_t^{\frac{1}{\sigma-1}} \left(\hat{K}_t^p\right)^{\alpha} (N_t^p)^{1-\alpha} \tag{A.20}$$

$$\hat{\tilde{w}}_t^u = (1-\alpha)\frac{\sigma-1}{\sigma} Z_t M_t^{\frac{1}{\sigma-1}} \left(\hat{K}_t^p\right)^{\alpha} (N_t^p)^{-\alpha} / \left(1+\zeta r_t^l\right) \tag{A.21}$$

$$r_t^k = \alpha\frac{\sigma-1}{\sigma} Z_t M_t^{\frac{1}{\sigma-1}} \left(\hat{K}_t^p\right)^{\alpha-1} (N_t^p)^{1-\alpha} / \left(1+\zeta r_t^l\right) \tag{A.22}$$

Households

$$1 = \mathrm{E}_t\left[\beta\left(g_t^Q \cdot \hat{C}_{t+1}/\hat{C}_t\right)^{-\theta}\left(1+r_{t+1}^d\right)\right] \tag{A.23}$$

$$\Lambda_{t,t+1} = \mathrm{E}_t\left[\left(g_t^Q \cdot \hat{C}_{t+1}/\hat{C}_t\right)^{-\theta}\right] \tag{A.24}$$

Financial system

$$r_t^l = sp_t + r_t^d \tag{A.25}$$

$$r_t^l = r_t^k - dp \tag{A.26}$$

Frictional labor markets (notation $w_t^* \equiv w_t(r)$)

$$m_t^u = \sigma_m (u_t^u)^\psi (v_t^u)^{1-\psi} \tag{A.27}$$
$$m_t^s = \sigma_m (u_t^s)^\psi (v_t^s)^{1-\psi} \tag{A.28}$$
$$n_{t+1}^u = (\rho^u + x_t^u) n_t^u \tag{A.29}$$
$$n_{t+1}^s = (\rho^s + x_t^s) n_t^s \tag{A.30}$$
$$u_t^u = 1 - n_t^u \tag{A.31}$$
$$u_t^s = 1 - n_t^s \tag{A.32}$$
$$q_t^u = m_t^u / v_t^u \tag{A.33}$$
$$q_t^s = m_t^s / v_t^s \tag{A.34}$$
$$p_t^u = m_t^u / u_t^u \tag{A.35}$$
$$p_t^s = m_t^s / u_t^s \tag{A.36}$$
$$x_t^u = q_t^u v_t^u / n_t^u \tag{A.37}$$
$$x_t^s = q_t^s v_t^s / n_t^s \tag{A.38}$$

$$\kappa^u x_t^u = \mathrm{E}_t \left[\beta \Lambda_{t,t+1} \left(\hat{\bar{w}}_{t+1}^u - \hat{w}_t^u + \frac{\kappa^u}{2} \left(x_{t+1}^u \right)^2 + \rho^u \kappa^u x_{t+1}^u \right) \right] \tag{A.39}$$
$$\kappa^s x_t^s = \mathrm{E}_t \left[\beta \Lambda_{t,t+1} \left(\hat{\bar{w}}_{t+1}^s - \hat{w}_t^s + \frac{\kappa^s}{2} \left(x_{t+1}^s \right)^2 + \rho^s \kappa^s x_{t+1}^s \right) \right] \tag{A.40}$$
$$\kappa^u x_t^{u*} = \mathrm{E}_t \left[\beta \Lambda_{t,t+1} \left(\hat{\bar{w}}_{t+1}^u - \hat{w}_t^{u*} + \frac{\kappa^u}{2} \left(x_{t+1}^{u*} \right)^2 + \rho^u \kappa^u x_{t+1}^{u*} \right) \right] \tag{A.41}$$
$$\kappa^s x_t^{s*} = \mathrm{E}_t \left[\beta \Lambda_{t,t+1} \left(\hat{\bar{w}}_{t+1}^s - \hat{w}_t^{s*} + \frac{\kappa^s}{2} \left(x_{t+1}^{s*} \right)^2 + \rho^s \kappa^s x_{t+1}^{s*} \right) \right] \tag{A.42}$$
$$\Delta_t^u = 1 + \beta \rho^u \lambda \mathrm{E}_t \left[\Lambda_{t,t+1} g_t^Q \Delta_{t+1}^u \right] \tag{A.43}$$
$$\Delta_t^s = 1 + \beta \rho^s \lambda \mathrm{E}_t \left[\Lambda_{t,t+1} g_t^Q \Delta_{t+1}^s \right] \tag{A.44}$$
$$\Delta_t^u \hat{w}_t^{u*} = \hat{w}_t^{uo} + \rho^u \lambda \mathrm{E}_t \left[\beta \Lambda_{t,t+1} \Delta_{t+1}^u \hat{w}_{t+1}^{u*} \right] \tag{A.45}$$
$$\Delta_t^s \hat{w}_t^{s*} = \hat{w}_t^{so} + \rho^s \lambda \mathrm{E}_t \left[\beta \Lambda_{t,t+1} \Delta_{t+1}^s \hat{w}_{t+1}^{s*} \right] \tag{A.46}$$
$$\hat{w}_t^{uf} = \psi \left(\hat{\bar{w}}_t^u + \frac{\kappa^u}{2} (x_t^u)^2 + p_t^u \kappa^u x_t^u \right) + (1-\psi) b_t^u \tag{A.47}$$
$$\hat{w}_t^{sf} = \psi \left(\hat{\bar{w}}_t^s + \frac{\kappa^s}{2} (x_t^s)^2 + p_t^s \kappa^s x_t^s \right) + (1-\psi) b_t^s \tag{A.48}$$
$$\begin{aligned} \hat{w}_t^{uo} &= \hat{w}_t^{uf} + \psi \left(\frac{\kappa^u}{2} \left((x_t^{u*})^2 - (x_t^u)^2 \right) + p_t^u \kappa^u \left(x_t^{u*} - x_t^u \right) \right) \\ &\quad + (1-\psi) p_t^u \mathrm{E}_t \left[\beta \Lambda_{t,t+1} \lambda \Delta_{t+1}^u g_t^Q \left(\hat{w}_t^u - \hat{w}_t^{u*} \right) \right] \end{aligned} \tag{A.49}$$
$$\begin{aligned} \hat{w}_t^{so} &= \hat{w}_t^{sf} + \psi \left(\frac{\kappa^s}{2} \left((x_t^{s*})^2 - (x_t^s)^2 \right) + p_t^s \kappa^s \left(x_t^{s*} - x_t^s \right) \right) \\ &\quad + (1-\psi) p_t^s \mathrm{E}_t \left[\beta \Lambda_{t,t+1} \lambda \Delta_{t+1}^s g_t^Q \left(\hat{w}_t^s - \hat{w}_t^{s*} \right) \right] \end{aligned} \tag{A.50}$$

$$\hat{w}_t^u = \lambda \hat{w}_{t-1}^u + (1-\lambda)\,\hat{w}_t^{u*} \tag{A.51}$$
$$\hat{w}_t^s = \lambda \hat{w}_{t-1}^s + (1-\lambda)\,\hat{w}_t^{s*} \tag{A.52}$$
$$\hat{b}_t^u = 0.4\hat{w}_{ss}^u \tag{A.53}$$
$$\hat{b}_t^s = 0.4\hat{w}_{ss}^s \tag{A.54}$$

Market clearing

$$\hat{Y}_t = \hat{C}_t + \hat{I}_t + \kappa^u \left(x_t^u\right)^2 N_t^p + \kappa^s \left(x_t^s\right)^2 N_t^s \tag{A.55}$$
$$g_t^Q \hat{K}_{t+1} = (1-dp)\,\hat{K}_t + \hat{I}_t \tag{A.56}$$
$$\hat{K}_t = \hat{K}_t^p + \hat{K}_t^s \tag{A.57}$$
$$N_t^p = (1-s)\,n_t^u \tag{A.58}$$
$$N_t^s = sn_t^s \tag{A.59}$$

Shocks

$$\log Z_t = \rho_Z \log Z_{t-1} + \varepsilon_{Z,t} \tag{A.60}$$
$$\log sp_t = (1-\rho_{sp}) \log sp_{ss} + \rho_{sp} \log sp_{t-1} + \varepsilon_{sp,t} \tag{A.61}$$

Welfare

$$U_t = \frac{\left(\hat{C}_t Q_t\right)^{1-\theta}}{1-\theta} + \beta \mathrm{E}_t\left[U_{t+1}\right] \tag{A.62}$$

A.3 Additional derivations

A.3.1 Solutions of cost minimization problems

Intermediate goods production sector

$$\begin{aligned} \min \quad & tc_t^p(i) = \left(1 + \zeta r_t^l\right)\left(\tilde{w}_t^u n_t^p(i) + r_t k_t^p(i)\right) \\ \text{subject to} \quad & y_t(i) = Z_t k_t^p(i)^\alpha \left[q_t(i) n_t^p(i)\right]^{1-\alpha} \end{aligned}$$

FOCs

$$\begin{aligned} n_t(i) \quad &: \quad \left(1 + \zeta r_t^l\right)\tilde{w}_t^u = \lambda^p (1-\alpha) Z_t k_t^p(i)^\alpha q_t(i)^{1-\alpha} n_t^p(i)^{-\alpha} \\ k_t(i) \quad &: \quad \left(1 + \zeta r_t^l\right) r_t = \lambda^p \alpha Z_t k_t^p(i)^{\alpha-1} q_t(i)^{1-\alpha} n_t^p(i)^{1-\alpha} \end{aligned}$$

Divide

$$\begin{aligned} \frac{\tilde{w}_t^u}{r_t} &= \frac{1-\alpha}{\alpha}\frac{k_t^p(i)}{n_t^p(i)} \\ k_t^p(i) &= \frac{\alpha}{1-\alpha}\frac{\tilde{w}_t^u}{r_t} n_t^p(i) \\ n_t^p(i) &= \frac{1-\alpha}{\alpha}\frac{r_t}{\tilde{w}_t^u} k_t^p(i) \end{aligned}$$

Production function

$$y_t(i) = Z_t k_t^p(i)^\alpha \left[q_t(i) n_t^p(i)\right]^{1-\alpha} = Z_t k_t^p(i)^\alpha \left[q_t(i)\frac{1-\alpha}{\alpha}\frac{r_t}{\tilde{w}_t^u} k_t^p(i)\right]^{1-\alpha} = Z_t k_t^p(i)\left[q_t(i)\frac{1-\alpha}{\alpha}\frac{r_t}{\tilde{w}_t^u}\right]^{1-\alpha}$$

$$k_t^p(i) = \frac{y_t(i)}{Z_t}\left[q_t(i)\frac{1-\alpha}{\alpha}\frac{r_t}{\tilde{w}_t^u}\right]^{\alpha-1}$$

Total cost

$$\begin{aligned} tc_t^p(i) &= \left(1 + \zeta r_t^l\right)\left(\tilde{w}_t^u n_t^p(i) + r_t k_t^p(i)\right) = \left(1 + \zeta r_t^l\right)\left(\tilde{w}_t^u \frac{1-\alpha}{\alpha}\frac{r_t}{\tilde{w}_t^u} k_t^p(i) + r_t k_t^p(i)\right) \\ &= \left(1 + \zeta r_t^l\right)\left(\frac{1-\alpha}{\alpha} + 1\right) r_t k_t^p(i) = \left(1 + \zeta r_t^l\right)\frac{r_t}{\alpha} k_t^p(i) \\ &= \left(1 + \zeta r_t^l\right)\frac{r_t}{\alpha}\frac{y_t(i)}{Z_t}\left[q_t(i)\frac{1-\alpha}{\alpha}\frac{r_t}{\tilde{w}_t^u}\right]^{\alpha-1} = \left(1 + \zeta r_t^l\right)\frac{y_t(i)}{Z_t}\left(\frac{r_t}{\alpha}\right)^\alpha \left(\frac{\tilde{w}_t^u / q_t(i)}{1-\alpha}\right)^{1-\alpha} \end{aligned}$$

Real marginal cost

$$mc_t^p(i) = \frac{\left(1 + \zeta r_t^l\right)}{Z_t}\left(\frac{r_t}{\alpha}\right)^\alpha \left(\frac{\tilde{w}_t^u / q_t(i)}{1-\alpha}\right)^{1-\alpha}$$

Research and development sector

$$\begin{aligned} \min \quad & tc_t^x(i) = \left(1+\zeta r_t^l\right)\left(\tilde{w}_t^s n_t^x(i) + r_t k_t^x(i)\right) \\ \text{subject to} \quad & x_t(i) = \frac{k_t^x(i)^\alpha \left[Q_t n_t^x(i)\right]^{1-\alpha}}{Q_t \phi_t(i)} \end{aligned}$$

FOCs

$$\begin{aligned} n_t^x(i) &: \left(1+\zeta r_t^l\right)\tilde{w}_t^s = \lambda(1-\alpha)\frac{Z_t k_t^x(i)^\alpha Q_t^{1-\alpha} n_t^x(i)^{-\alpha}}{Q_t\phi_t(i)} \\ k_t^x(i) &: \left(1+\zeta r_t^l\right) r_t = \lambda\alpha\frac{Z_t k_t^x(i)^{\alpha-1} Q_t^{1-\alpha} n_t^x(i)^{1-\alpha}}{Q_t\phi_t(i)} \end{aligned}$$

Divide

$$\begin{aligned} \frac{\tilde{w}_t^s}{r_t} &= \frac{1-\alpha}{\alpha}\frac{k_t^x(i)}{n_t^x(i)} \\ k_t^x(i) &= \frac{\alpha}{1-\alpha}\frac{\tilde{w}_t^s}{r_t} n_t^x(i) \\ n_t^x(i) &= \frac{1-\alpha}{\alpha}\frac{r_t}{\tilde{w}_t^s} k_t^x(i) \end{aligned}$$

R&D production function

$$x_t(i) = \frac{k_t^x(i)^\alpha \left[Q_t n_t^x(i)\right]^{1-\alpha}}{Q_t\phi_t(i)} = Q_t^{-\alpha} k_t^x(i)\left(\frac{1-\alpha}{\alpha}\frac{r_t}{\tilde{w}_t^s}\right)^{1-\alpha} / \phi_t(i)$$

$$k_t^x(i) = x_t(i)\, Q_t^\alpha \left(\frac{1-\alpha}{\alpha}\frac{r_t}{\tilde{w}_t^s}\right)^{\alpha-1}\phi_t(i)$$

Total cost

$$\begin{aligned} tc_t^x(i) &= \left(1+\zeta r_t^l\right)\frac{r_t}{\alpha}k_t^x(i) = \left(1+\zeta r_t^l\right)\frac{r_t}{\alpha}x_t(i)\,Q_t^\alpha\left(\frac{1-\alpha}{\alpha}\frac{r_t}{\tilde{w}_t^s}\right)^{\alpha-1}\phi_t(i) \\ &= \left(1+\zeta r_t^l\right)x_t(i)\,Q_t^\alpha\left(\frac{r_t}{\alpha}\right)^\alpha\left(\frac{\tilde{w}_t^s}{1-\alpha}\right)^{1-\alpha}\phi_t(i) \end{aligned}$$

Real marginal cost

$$mc_t^x(i) = \left(1+\zeta r_t^l\right)Q_t^\alpha\left(\frac{r_t}{\alpha}\right)^\alpha\left(\frac{\tilde{w}_t^s}{1-\alpha}\right)^{1-\alpha}\phi_t(i) \equiv \bar{mc}_t^x\phi_t(i)$$

Total cost as function of desired innovative success probability

$$\begin{aligned} \chi_t(i) &= \frac{a x_t(i)}{1+a x_t(i)} \\ x_t(i) &= \frac{1}{a}\frac{\chi_t(i)}{1-\chi_t(i)} \\ tc_t^x(i) &= \left(1+\zeta r_t^l\right)\frac{\bar{mc}_t^x}{a}\frac{\chi_t(i)}{1-\chi_t(i)}\phi_t(i) \end{aligned}$$

A.3.2 Aggregate production function

Relative inputs

$$\frac{y_t(i)}{y_t(j)} = \frac{Y_t p_t(i)^{-\sigma}}{Y_t p_t(j)^{-\sigma}} = \left[\frac{\frac{\sigma}{\sigma-1}\frac{\left(1+\zeta r_t^l\right)}{Z_t}\left(\frac{r_t}{\alpha}\right)^{\alpha}\left(\frac{\tilde{w}_t^u/q_t(i)}{1-\alpha}\right)^{1-\alpha}}{\frac{\sigma}{\sigma-1}\frac{\left(1+\zeta r_t^l\right)}{Z_t}\left(\frac{r_t}{\alpha}\right)^{\alpha}\left(\frac{\tilde{w}_t^u/q_t(j)}{1-\alpha}\right)^{1-\alpha}}\right]^{-\sigma} = \left(\frac{q_t(i)^{\alpha-1}}{q_t(j)^{\alpha-1}}\right)^{-\sigma} = \left(\frac{q_t(i)^{1-\alpha}}{q_t(j)^{1-\alpha}}\right)^{\sigma}$$

$$\frac{y_t(i)}{y_t(j)} = \frac{Z_t k_t^p(i)\left[q_t(i)\,\frac{1-\alpha}{\alpha}\frac{r_t}{\tilde{w}_t^u}\right]^{1-\alpha}}{Z_t k_t^p(j)\left[q_t(j)\,\frac{1-\alpha}{\alpha}\frac{r_t}{\tilde{w}_t^u}\right]^{1-\alpha}}$$

$$\frac{k_t^p(i)\,q_t(i)^{1-\alpha}}{k_t^p(j)\,q_t(j)^{1-\alpha}} = \left(\frac{q_t(i)^{1-\alpha}}{q_t(j)^{1-\alpha}}\right)^{\sigma}$$

$$\frac{k_t^p(i)}{k_t^p(j)} = \left(\frac{q_t(i)}{q_t(j)}\right)^{(1-\alpha)(\sigma-1)}$$

$$k_t^p(i) = \left(\frac{q_t(i)}{q_t(j)}\right)^{(1-\alpha)(\sigma-1)} k_t^p(j)$$

$$k_t^p(i) = \left(\frac{q_t(i)}{Q_t}\right)^{(1-\alpha)(\sigma-1)} \bar{k}_t^p$$

$$n_t^p(i) = \left(\frac{q_t(i)}{Q_t}\right)^{(1-\alpha)(\sigma-1)} \bar{n}_t^p$$

where $\bar{k}_t^p \equiv K_t^p/M_t$ and $\bar{n}_t^p \equiv N_t^p/M_t$.

Final goods output

$$\begin{aligned}
Y_t &= \left[\int_0^{M_t} y_t(i)^{\frac{\sigma-1}{\sigma}}\,\mathrm{d}i\right]^{\frac{\sigma}{\sigma-1}} = \left[M_t\int_0^{\infty} y_t(q)^{\frac{\sigma-1}{\sigma}}\,\mu_t(q)\,\mathrm{d}q\right]^{\frac{\sigma}{\sigma-1}} \\
&= M_t^{\frac{\sigma}{\sigma-1}}\left[\int_0^{\infty}\left[Z_t k_t^p(q)^{\alpha}\,q^{1-\alpha} n_t^p(q)^{1-\alpha}\right]^{\frac{\sigma-1}{\sigma}}\mu_t(q)\,\mathrm{d}q\right]^{\frac{\sigma}{\sigma-1}} \\
&= M_t^{\frac{\sigma}{\sigma-1}} Z_t\left[\int_0^{\infty}\left[\left(\frac{q}{Q_t}\right)^{(1-\alpha)(\sigma-1)}\left(\bar{k}_t^p\right)^{\alpha}\left(\bar{n}_t^p\right)^{1-\alpha} q^{1-\alpha}\right]^{\frac{\sigma-1}{\sigma}}\mu_t(q)\,\mathrm{d}q\right]^{\frac{\sigma}{\sigma-1}} \\
&= M_t^{\frac{\sigma}{\sigma-1}} Z_t\left(\bar{k}_t^p\right)^{\alpha}\left(\bar{n}_t^p\right)^{1-\alpha} Q_t^{(1-\alpha)(1-\sigma)}\left[\int_0^{\infty}\left[\left(q^{1-\alpha}\right)^{\sigma}\right]^{\frac{\sigma-1}{\sigma}}\mu_t(q)\,\mathrm{d}q\right]^{\frac{\sigma}{\sigma-1}} \\
&= M_t^{\frac{\sigma}{\sigma-1}-1} Z_t\left(K_t^p\right)^{\alpha}\left(N_t^p\right)^{1-\alpha} Q_t^{(1-\alpha)(1-\sigma)}\left[\left[\int_0^{\infty}\left(q^{1-\alpha}\right)^{\sigma-1}\mu_t(q)\,\mathrm{d}q\right]^{\frac{1}{\sigma-1}}\right]^{\sigma} \\
&= M_t^{\frac{1}{\sigma-1}} Z_t\left(K_t^p\right)^{\alpha}\left(N_t^p\right)^{1-\alpha} Q_t^{(1-\alpha)(1-\sigma)}\left(Q_t^{1-\alpha}\right)^{\sigma} \\
&= M_t^{\frac{1}{\sigma-1}} Z_t\left(K_t^p\right)^{\alpha}\left(Q_t N_t^p\right)^{1-\alpha}
\end{aligned}$$

A.3.3 Real profit function

Real operating profit

$$\pi_t^o(i) = p_t(i)\, y_t(i) - mc_t^p(i)\, y_t(i) - f_t = p_t(i)\, y_t(i) - p_t(i)\frac{\sigma-1}{\sigma} y_t(i) - f_t$$

$$= \left(1 - \frac{\sigma-1}{\sigma}\right) Y_t p_t(i)^{1-\sigma} - f_t = \frac{1}{\sigma} Y_t \left[\frac{\sigma}{\sigma-1}\frac{\left(1+\zeta r_t^l\right)}{Z_t}\left(\frac{r_t}{\alpha}\right)^{\alpha}\left(\frac{\tilde{w}_t^u/q_t(i)}{1-\alpha}\right)^{1-\alpha}\right]^{1-\sigma} - f_t$$

$$= \frac{(\sigma-1)^{\sigma-1}}{\sigma^\sigma} Y_t Z_t^{\sigma-1}\left[\left(1+\zeta r_t^l\right)\left(\frac{r_t}{\alpha}\right)^{\alpha}\left(\frac{\tilde{w}_t^u/q_t(i)}{1-\alpha}\right)^{1-\alpha}\right]^{1-\sigma} - f_t$$

Price index (where $R_t \equiv P_t r_t$ and $W_t^u \equiv P_t \tilde{w}_t^u$)

$$P_t = \left[\int_0^{M_t} P_t(i)^{1-\sigma}\,\mathrm{d}i\right]^{\frac{1}{1-\sigma}} = \left[M_t \int_0^\infty P_t(q)^{1-\sigma}\mu_t(q)\,\mathrm{d}q\right]^{\frac{1}{1-\sigma}}$$

$$= M_t^{\frac{1}{1-\sigma}}\left[\int_0^\infty \left[\frac{\sigma}{\sigma-1}\frac{\left(1+\zeta r_t^l\right)}{Z_t}\left(\frac{R_t}{\alpha}\right)^{\alpha}\left(\frac{W_t^u/q}{1-\alpha}\right)^{1-\alpha}\right]^{1-\sigma}\mu_t(q)\,\mathrm{d}q\right]^{\frac{1}{1-\sigma}}$$

$$= \frac{\sigma}{\sigma-1} M_t^{\frac{1}{1-\sigma}}\frac{\left(1+\zeta r_t^l\right)}{Z_t}\left(\frac{R_t}{\alpha}\right)^{\alpha}\left(\frac{W_t^u}{1-\alpha}\right)^{1-\alpha}\left[\int_0^\infty \left(q^{\alpha-1}\right)^{1-\sigma}\mu_t(q)\,\mathrm{d}q\right]^{\frac{1}{1-\sigma}}$$

$$= \frac{\sigma}{\sigma-1} M_t^{\frac{1}{1-\sigma}}\frac{\left(1+\zeta r_t^l\right)}{Z_t}\left(\frac{R_t}{\alpha}\right)^{\alpha}\left(\frac{W_t^u}{1-\alpha}\right)^{1-\alpha}\left[\left[\int_0^\infty \left(q^{1-\alpha}\right)^{\sigma-1}\mu_t(q)\,\mathrm{d}q\right]^{\frac{1}{\sigma-1}}\right]^{-1}$$

$$= \frac{\sigma}{\sigma-1} M_t^{\frac{1}{1-\sigma}}\frac{\left(1+\zeta r_t^l\right)}{Z_t}\left(\frac{R_t}{\alpha}\right)^{\alpha}\left(\frac{W_t^u}{1-\alpha}\right)^{1-\alpha}\left(Q_t^{1-\alpha}\right)^{-1}$$

$$= \frac{\sigma}{\sigma-1} M_t^{\frac{1}{1-\sigma}}\frac{\left(1+\zeta r_t^l\right)}{Z_t}\left(\frac{R_t}{\alpha}\right)^{\alpha}\left(\frac{W_t^u/Q_t}{1-\alpha}\right)^{1-\alpha}$$

Real input cost index

$$\left(1+\zeta r_t^l\right)\left(\frac{R_t}{\alpha}\right)^{\alpha}\left(\frac{W_t^u/Q_t}{1-\alpha}\right)^{1-\alpha} = \frac{\sigma-1}{\sigma} P_t M_t^{\frac{1}{\sigma-1}} Z_t$$

$$\left(1+\zeta r_t^l\right)\left(\frac{r_t}{\alpha}\right)^{\alpha}\left(\frac{\tilde{w}_t^u}{1-\alpha}\right)^{1-\alpha} = \frac{\sigma-1}{\sigma} M_t^{\frac{1}{\sigma-1}} Z_t Q_t^{1-\alpha}$$

Real operating profit

$$\begin{aligned}
\pi_t^o(i) &= \frac{(\sigma-1)^{\sigma-1}}{\sigma^\sigma} Y_t Z_t^{\sigma-1} \left[\left(1+\zeta r_t^l\right) \left(\frac{r_t}{\alpha}\right)^\alpha \left(\frac{w_t/q_t(i)}{1-\alpha}\right)^{1-\alpha} \right]^{1-\sigma} - f_t \\
&= \frac{(\sigma-1)^{\sigma-1}}{\sigma^\sigma} Y_t Z_t^{\sigma-1} \left[\frac{\sigma-1}{\sigma} M_t^{\frac{1}{\sigma-1}} Z_t Q_t^{1-\alpha} q_t(i)^{\alpha-1}\right]^{1-\sigma} - f_t \\
&= \frac{Y_t}{\sigma M_t} \left[\left(\frac{q_t(i)}{Q_t}\right)^{1-\alpha}\right]^{\sigma-1} - f_t \\
&= \frac{Y_t}{\sigma M_t} \phi_t(i) - f_t
\end{aligned}$$

Real profit

$$\begin{aligned}
\pi_t(i) &= \pi_t^o(i) - \left(1+\zeta r_t^l\right) \frac{\bar{mc}_t^x}{a} \frac{\chi_t(i)}{1-\chi_t(i)} \phi_t(i) \\
&= \left(\frac{Y_t}{\sigma M_t} - \left(1+\zeta r_t^l\right) \frac{\bar{mc}_t^x}{a} \frac{\chi_t(i)}{1-\chi_t(i)}\right) \phi_t(i) - f_t \\
&= \left(\frac{Y_t}{\sigma M_t} - \left(1+\zeta r_t^l\right) \frac{\bar{mc}_t^x}{a} \frac{\chi_t(i)}{1-\chi_t(i)}\right) \phi_t(i) - \left(1+\zeta r_t^l\right) \bar{mc}_t^x f \\
&= Y_t \left[\left(\frac{1}{\sigma M_t} - \left(1+\zeta r_t^l\right) \frac{\omega_t}{a} \frac{\chi_t(i)}{1-\chi_t(i)}\right) \phi_t(i) - \left(1+\zeta r_t^l\right) \omega_t f\right]
\end{aligned}$$

A.3.4 Evolution of aggregate quality index

Following Melitz (2003), I consider the current period distribution of quality levels $\mu_t(q)$ to be a truncated part of an underlying distribution $g_t(q)$, so that:

$$\mu_t(q) = \begin{cases} 1/\left[1 - G_t\left(q_{t-1}^*\right)\right] g_t(q) & \text{if } q \geq q_{t-1}^* \\ 0 & \text{otherwise} \end{cases}$$

where $q_t^* = (\phi_t^*)^{1/[(1-\alpha)(\sigma-1)]} Q_t$.

Aggregate quality index at the end of period t:

$$Q_t^{1-\alpha} = \left[\int_0^\infty \left(q^{1-\alpha}\right)^{\sigma-1} \mu_t(q)\,\mathrm{d}q\right]^{\frac{1}{\sigma-1}} = \left[\frac{1}{1-G_t(q_{t-1}^*)} \int_{q_{t-1}^*}^\infty \left(q^{1-\alpha}\right)^{\sigma-1} g_t(q)\,\mathrm{d}q\right]^{\frac{1}{\sigma-1}}$$

The aggregate quality level after exits and innovation resolution but before entry:

$$\begin{aligned}
Q_t^* &= \left\{\frac{1}{1-G_t(q_t^*)} \left[(1-\chi_t) \int_{q_t^*}^\infty \left(q^{1-\alpha}\right)^{\sigma-1} g_t(q)\,\mathrm{d}q + \chi_t \int_{q_t^*}^\infty \left(\iota^{\frac{1}{(1-\alpha)(\sigma-1)}} q\right)^{(1-\alpha)(\sigma-1)} g_t(q)\,\mathrm{d}q\right]\right\}^{\frac{1}{\sigma-1}} \\
&= \left[(1-\chi_t+\chi_t\iota) \frac{1}{1-G_t(q_t^*)} \int_{q_t^*}^\infty \left(q^{1-\alpha}\right)^{\sigma-1} g_t(q)\,\mathrm{d}q\right]^{\frac{1}{\sigma-1}}
\end{aligned}$$

Aggregate quality index in $t+1$ after entry:

$$Q_{t+1}=\left\{\frac{1-\chi_t+\chi_t\iota}{1-G_t\left(q_t^*\right)}\left[\begin{array}{c}\left(1-\frac{M_t^e}{M_{t+1}}\right)\int_{q_t^*}^{\infty}\left(q^{1-\alpha}\right)^{\sigma-1}g_t(q)\mathrm{d}q\\+\frac{M_t^e}{M_{t+1}}\int_{q_t^*}^{\infty}\left(\left(\frac{\sigma}{\sigma-1}\right)^{\frac{1}{(1-\alpha)(\sigma-1)}}q\right)^{(1-\alpha)(\sigma-1)}g_t(q)\mathrm{d}q\end{array}\right]\right\}^{\frac{1}{\sigma-1}}$$

$$=\left[\left(1-\chi_t+\chi_t\iota\right)\left(1-\frac{M_t^e}{M_{t+1}}+\frac{M_t^e}{M_{t+1}}\frac{\sigma}{\sigma-1}\right)\frac{1}{1-G_t\left(q_t^*\right)}\int_{q_t^*}^{\infty}\left(q^{1-\alpha}\right)^{\sigma-1}g_t\left(q\right)\mathrm{d}q\right]^{\frac{1}{\sigma-1}}$$

Transformed aggregate growth rate η_t:

$$\begin{aligned}\eta_t &= \left(\frac{Q_{t+1}}{Q_t}\right)^{(1-\alpha)(\sigma-1)}\\
&= \left\{\frac{\left[\left(1-\chi_t+\chi_t\iota\right)\left(1-\frac{M_t^e}{M_{t+1}}+\frac{M_t^e}{M_{t+1}}\frac{\sigma}{\sigma-1}\right)\frac{1}{1-G_t\left(q_t^*\right)}\int_{q_t^*}^{\infty}\left(q^{1-\alpha}\right)^{\sigma-1}g_t\left(q\right)\mathrm{d}q\right]^{\frac{1}{\sigma-1}}}{\left[\frac{1}{1-G_t\left(q_{t-1}^*\right)}\int_{q_{t-1}^*}^{\infty}\left(q^{1-\alpha}\right)^{\sigma-1}g_t\left(q\right)\mathrm{d}q\right]^{\frac{1}{\sigma-1}}}\right\}^{\sigma-1}\\
&\approx \left(1-\chi_t+\chi_t\iota\right)\left(1-\frac{M_t^e}{M_{t+1}}+\frac{M_t^e}{M_{t+1}}\frac{\sigma}{\sigma-1}\right)\end{aligned}$$

where if the distribution is invariant with respect to the cutoff points q_{t-1}^* and q_t^* (as is the case with Pareto and other power-law distributions) then the above relationship holds with equality.

A.3.5 Wages

Denote by $W_t\left(j\right)$ the expected discounted sum of future wages received over the duration of the relationship with the employment agency:

$$W_t\left(j\right)=\Delta_t w_t\left(j\right)+\left(1-\lambda\right)\mathrm{E}_t\sum_{s=1}^{\infty}\left(\beta\rho\right)^s\Lambda_{t,t+s}\Delta_{t+s}w_{t+s}\left(r\right)$$

where the first part represents contract that is not renegotiated and the wage is only indexed, while the second part represents future, renegotiated contracts at the same employment agency, and:

$$\Delta_t=\mathrm{E}_t\sum_{s=0}^{\infty}\left(\beta\rho\lambda\right)^s\Lambda_{t,t+s}\frac{Q_{t+s}}{Q_t}$$

The surplus of workers at renegotiating agency can be then rewritten as:

$$\begin{aligned}H_t\left(r\right)&=w_t\left(r\right)+\mathrm{E}_t\left[\beta\Lambda_{t,t+1}\rho H_{t+1}\left(r\right)\right]-b_t-\mathrm{E}_t\left[\beta\Lambda_{t,t+1}p_tH_{t+1}\right]\\
&=W_t\left(r\right)-\mathrm{E}_t\sum_{s=0}^{\infty}\left(\beta\rho\right)^s\Lambda_{t,t+s}\left(b_{t+s}+p_{t+s}H_{t+s+1}\right)\end{aligned}$$

Similarly, the surplus value of employed worker from the point of view of the employment agency can be rewritten as:

$$J_t(r) = \tilde{w}_t + \frac{\kappa}{2} x_t^2(r) + \rho \mathrm{E}_t\left[\beta \Lambda_{t,t+1} J_{t+1}(r)\right] - w_t(r)$$
$$= \mathrm{E}_t \sum_{s=0}^{\infty} (\beta\rho)^s \Lambda_{t,t+s} \left(\tilde{w}_{t+s} + \frac{\kappa}{2} x_{t+s}^2(r)\right) - W_t(r)$$

By substituting the above expressions in the surplus sharing equation one can obtain:

$$W_t(r) = \psi \mathrm{E}_t \sum_{s=0}^{\infty} (\beta\rho)^s \Lambda_{t,t+s} \left(\tilde{w}_{t+s} + \frac{\kappa}{2} x_{t+s}^2(r)\right)$$
$$+ (1-\psi)\mathrm{E}_t \sum_{s=0}^{\infty} (\beta\rho)^s \Lambda_{t,t+s} \left(b_{t+s} + p_{t+s} H_{t+s+1}\right)$$

or, after simplifying, in the following recursive form:

$$\Delta_t w_t(r) = \psi \left(\tilde{w}_t + \frac{\kappa}{2} x_t^2(r)\right) + (1-\psi)\left(b_t + p_t \mathrm{E}_t\left[\beta \Lambda_{t,t+1} H_{t+1}\right]\right)$$
$$+ \rho\lambda \mathrm{E}_t\left[\beta \Lambda_{t,t+1} \Delta_{t+1} w_{t+1}(r)\right]$$

where the first two terms comprise the target wage w_t^o, which in turn can be expressed in relation to the flexible contract wage:

$$w_t^o = \psi \left(\tilde{w}_t + \frac{\kappa}{2} x_t^2(r)\right) + (1-\psi)\left(b_t + p_t \mathrm{E}_t\left[\beta \Lambda_{t,t+1} H_{t+1}\right]\right)$$
$$= w_t^f + \psi \left(\frac{\kappa}{2}\left(x_t^2(r) - x_t^2\right) - p_t \kappa x_t\right) + (1-\psi) p_t E_t\left[\beta \Lambda_{t,t+1} H_{t+1}\right]$$

Average vs conditional on renegotiation worker surplus

$$H_t = H_t(r) + \Delta_t\left(w_t - w_t(r)\right)$$

Therefore

$$(1-\psi) p_t E_t\left[\beta \Lambda_{t,t+1} H_{t+1}\right] =$$
$$= (1-\psi) p_t E_t\left[\beta \Lambda_{t,t+1}\left[H_{t+1}(r) + \lambda \Delta_{t+1}\left(w_{t+1} - w_{t+1}(r)\right)\right]\right]$$
$$= (1-\psi) p_t E_t\left[\beta \Lambda_{t,t+1} H_{t+1}(r)\right] + (1-\psi) p_t E_t\left[\beta \Lambda_{t,t+1} \lambda \Delta_{t+1}\left(w_{t+1} - w_{t+1}(r)\right)\right]$$
$$= \psi p_t E_t\left[\beta \Lambda_{t,t+1} J_{t+1}(r)\right] + (1-\psi) p_t E_t\left[\beta \Lambda_{t,t+1} \lambda \Delta_{t+1}\left(w_{t+1} - w_{t+1}(r)\right)\right]$$
$$= \psi p_t \kappa x_t(r) + (1-\psi) p_t E_t\left[\beta \Lambda_{t,t+1} \lambda \Delta_{t+1}\left(w_{t+1} - w_{t+1}(r)\right)\right]$$

Resulting target wage

$$w_t^o = w_t^f + \psi \left(\frac{\kappa}{2}\left(x_t^2(r) - x_t^2\right) + p_t \kappa\left(x_t(r) - x_t\right)\right)$$
$$+ (1-\psi) p_t \mathrm{E}_t\left[\beta \Lambda_{t,t+1} \lambda \Delta_{t+1}\left(w_{t+1} - w_{t+1}(r)\right)\right]$$

The above equation emphasizes the presence of spillovers of economy-wide wages on the bargaining wage. Intuitively, more intensive hiring by an agency requires also higher bargained wages, which are also upwardly pressured by the future average wage.

Finally, let x_t denote the average hiring rate:

$$x_t = \int_0^1 x_t(j) \frac{n_t(j)}{n_t} \mathrm{d}j$$

Then the job creation condition can be used to express x_t as:

$$\begin{aligned} \kappa x_t = {} & \mathrm{E}_t \left[\beta \Lambda_{t,t+1} \left(\tilde{w}_{t+1} - w_{t+1} + \frac{\kappa}{2} x_{t+1}^2 + \rho \kappa x_{t+1} \right) \right] \\ & + \mathrm{E}_t \left[\begin{array}{c} \beta \Lambda_{t,t+1} \int_0^1 \left(\frac{\kappa}{2} x_{t+1}^2(j) + \rho \kappa x_{t+1}(j) - w_{t+1}(j) \right) \frac{n_t(j)}{n_t} \mathrm{d}j \\ - \left(\frac{\kappa}{2} x_{t+1}^2 + \rho \kappa x_{t+1} - w_{t+1} \right) \end{array} \right] \end{aligned}$$

Note that along the balanced growth path the deviations of individual employment agencies' decisions from average disappear and as a first order approximation one can take only the first line of the above equation.