

Corporate Governance, Taxation and Business Cycles

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Abstract

We present an agency model of corporate tax auditing by a residual claimant government and embed it into a macro model with financial constraints. In our economy, entrepreneurs with access to risky investment technologies raise funds by issuing equity claims to new capital. Information asymmetries create incentives to choose a riskier but cheaper technology that provides private benefits and opportunities to evade taxes. Random auditing by the government for tax verification reveals technology choice, reducing the asymmetric information problem between lenders and borrowers. We show that moderate corporate governance quality accompanied by high taxes raise output, investment and consumption.

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1 Introduction

The classic principal–agent problem in corporate finance focuses on how to design contracts that align shareholders and managers interests. One way of solving the agency problem is to have some shareholders engage in the monitoring of management. In the credit rationing models of [Holmstrom and Tirole \(1997\)](#), the monitor can be a large shareholder who solves the asymmetric information problem at a cost. However, in the presence of taxation with the possibility of auditing, the asymmetric information problem can also be resolved by a government or tax authority that audits firms to determine if profits have been truthfully reported. We show in this paper that the possibility of auditing by the tax authority helps to reduce moral hazard in a setting where “entrepreneurs” are credit constrained. Our analysis is inspired by the tax claim a government has to firm profits. As noted by [Desai, Dyck and Zingales \(2007\)](#), the state can be thought of as the largest minority shareholder of most corporations due to its tax claim on profits. When there is an asymmetric information problem between lenders and borrowers, then the state, by occasionally auditing borrowing firms, provides the services of a typical “monitor” in the sense of models of managerial incentives or compensation ([Hanlon, Hoopes and Shroff, 2014](#); [Holmstrom and Tirole, 1997](#)).

We embed our agency problem into the DSGE model of financial frictions developed by [Kiyotaki and Moore \(2012\)](#). However, unlike the [Kiyotaki and Moore](#) model where the financial friction in the form of a credit constraint is calibrated from the data, our model is designed so that the level of the constraint is an equilibrium outcome. In our version of the [Kiyotaki and Moore](#) economy, the fraction of future earnings that an entrepreneur can pledge to lenders, the mortgageable fraction of equity, is a function of the quality of corporate governance, the tax system and the equilibrium equity price. We perform a detailed analysis of the principal agent problem that gives rise to the credit constraint and show how changes in governance institutions and tax rates have both a level and amplification effect on output, investment and asset prices.

In our economy, ex-ante identical entrepreneurs have access to two types of risky capital production technologies, a safer and riskier one. The riskier technology is less costly but provides an opportunity to extract private benefits: an entrepreneur who has used the riskier technology can claim the higher cost associated with the safer one when declaring profits. Corporate governance quality determines the size of the cost difference between the two technologies and hence the amount of funds that can be diverted as a pure private benefit to the entrepreneur. However, since using the higher cost in profit declaration is tax evasion, the entrepreneur faces a risk of discovery upon auditing by the tax authority. The audit would reveal her true cost and technology choice, both to the tax authority and lenders. Consequently, tax rates, audit frequencies and surcharges for evasion, together with corporate governance quality engendered in the level of private benefits will determine an entrepreneur’s incentive to choose between the two technologies. These choices determine how much funds can be raised to invest as well as the aggregate level of investment, capital and output. Moderate quality corporate governance accompanied by high tax rates increase investment, output and consumption while lowering asset prices and tax evasion. Low taxes or very low levels of private benefits have the opposite

effects.

The mechanism driving our results is as follows. When taxes and the penalties for tax evasion are low, entrepreneurs have a higher incentive to use riskier capital production technologies and enjoy the associated private benefits. Since investment is now more risky, entrepreneurs need to raise a higher value of internal funds in order to credibly signal their commitment to the safe technology, so the *mortgageable fraction of equity falls*. This results into a lower equilibrium aggregate capital level and consequently lower investment, output and consumption. When an economic shock occurs under low taxes, the fall in investment is deeper and the recovery slower as riskier capital production technologies imply additions to the aggregate stock of capital can only occur gradually. The converse is true under high taxes which discourage the use of risky technologies. When the quality of corporate governance institutions is high, entrepreneurs can mortgage a larger fraction of the future earnings, but this also encourages the use of the riskier technology, so that aggregate investment, consumption and output falls. Our model indicates that the best combination of taxes and private benefits involves high tax rates and moderate quality governance institutions where the mortgageable fraction of equity is lower and the probability of an entrepreneur using the safe technology is high. We now provide a brief survey of the literature that relates our work to empirical findings on corporate governance, tax auditing and the cost of credit.

Relation to the literature There is empirical evidence to support the notion that the ease of financing is related to the quality of corporate governance and the level of tax evasion. [Guedhami and Pittman \(2008\)](#) study how tax auditing affect yield spreads and find that debt financing is cheaper when a firm faces a higher probability of an IRS audit, with the impact larger for firms with concentrated ownership. The lower cost of funds for high audit probability firms arises from the reduction in information asymmetry between investors and the management (or controlling shareholders). The size of the economic effect is large: increasing IRS audit probability from 19% to 35% reduces the interest rate of a firm by on average 25 basis points. [Graham, Li and Qiu \(2008\)](#) study the effect of corporate misreporting and the subsequent financial restatements following audits, on bank loan contracting. They find that loans initiated after a restatement have higher spreads, shorter maturity and a higher likelihood of being secured. Similar findings have been made by [Karjalainen \(2011\)](#) and [Hasan, Hoi, Wu and Zhang \(2014\)](#). In a cross country analysis, [Beck, Lin and Ma \(2014\)](#) find that firms with better credit information sharing systems evade taxes to a lesser degree. [Artavanis, Morse and Tsoutsoura \(2016\)](#) use a bank-underwriting model to infer the level of tax evasion based on lending. The key innovation of their paper is the use of a bank's loan to income ratio, which should not be exceeded if the bank were to manage default risk. They show that in an environment characterised by tax evasion, banks use their private knowledge of true income in order to advance loans larger than the “book reported” loan-to-income ratio would allow.

Our analysis incorporates detailed micro-foundations on the origin of credit constraints in dynamic general equilibrium models, similar to those provided by [Chen \(2001\)](#); [Minetti \(2007\)](#) and [Bolton and Freixas \(2006\)](#). The first two authors use the continuous investment

environment of [Holmstrom and Tirole \(1997\)](#) in equilibrium models with a banking sector to show how credit constraints interact with lending to enhance the effects of negative productivity shocks. In [Bolton and Freixas](#), asymmetric information and information dilutions costs as in [Myers and Majluf \(1984\)](#) make it expensive for banks to raise capital and extend credit. More recently, [Benhabib, Dong and Wang \(2018\)](#) add a simple asymmetric information problem to a standard RBC type model. Their economy feature a fixed and endogenous measure of dishonest and honest borrowers, respectively. Their endogenous measure of dishonest borrowers is conceptually similar to our fraction of risky, possibly tax evading entrepreneurs.

Since the Great Recession of 2007, there has been a renewed interest in macroeconomic models that link liquidity to the severity of downturns. Recent work in this area include [Del Negro, Eggertsson, Ferrero and Kiyotaki \(2017\)](#) and [Bigio and Schneider \(2017\)](#), both based on [Kiyotaki and Moore](#)'s analysis of liquidity in a monetary economy. These papers include borrowing and funding constraints calibrated to match aggregate economic data. While our macroeconomic environment is similar, unlike these papers, the levels of credit constraints in our economy are an equilibrium outcome that derives from the quality of corporate governance and features of the government revenue system.

2 Model Economy

We first describe a macroeconomic environment based on a modified version of [Kiyotaki and Moore \(2012\)](#). In the model economy, entrepreneurs use their own skill and capital stock to produce output. Capital depreciates and is replenished through investment, but the investment technology, for producing new capital from output, is not commonly available: in each period, only a small fraction of the entrepreneurs are able to invest, and the arrival of investment opportunities is randomly distributed across entrepreneurs through time. As a result, there is need to shift savings from those without investment opportunities (savers/lenders/investors) to those with an opportunity (entrepreneurs/borrowers). We describe this in subsection [2.1](#). Borrowing and lending give rise to an agency problem which we detail in [2.2](#). We combine these two to determine the macroeconomic equilibrium in [2.3](#).

2.1 Macroeconomic Environment

Consider an infinite horizon discrete time economy with two objects traded: durable output and equity. At each time period, there are two types of agents, entrepreneurs and investors/savers. At date t , a typical entrepreneur's discounted utility is: $\sum_{j=0}^{\infty} \beta^j \log c_{t+j}$ where $\{c_t, c_{t+1}, \dots\}$ is a consumption path and $0 < \beta < 1$. All entrepreneurs have access to a decreasing returns to scale technology of producing output from capital. An entrepreneur holding $k_{j,t-1}$ units of capital can produce $\chi_t k_{j,t-1}^{\varphi}$ units of output where $\varphi \in (0, 1)$ and the productivity parameter $\chi_t > 0$ is common across all entrepreneurs and follows a stationary stochastic process. There is

a continuum of unit measure entrepreneurs and their aggregate output is given by:

$$Y_t = \left(\int_0^1 \chi_t k_{j,t-1}^\varphi dj \right)^{\frac{\alpha}{\varphi}} = \chi_t^{\frac{\alpha}{\varphi}} K_{t-1}^\alpha \quad (1)$$

where $\alpha \in (0, 1)$ and $K_t = \int_0^1 k_{t,j} dj$ is aggregate capital. Production is completed within the period t during which capital depreciates to $(1 - \delta)k_{t-1}$ for $\delta \in (0, 1)$. The return to capital r_t depends upon the productivity χ_t and the aggregate amount of capital.

The entrepreneur also has an opportunity to produce new capital. At each date t with probability π , she has an opportunity to produce i_t units of capital from some units of output. The arrival of such an opportunity is independently distributed across entrepreneurs and through time while also being independent of the productivity. Newly produced capital is available for production the next period: $k_t = (1 - \delta)k_{t-1} + i_t$. In order to finance the cost of investment, the entrepreneur who has an investment opportunity may issue equity claims to the future return of newly produced capital. One unit of equity is the claim to the return of one unit of investment: it produces r_{t+1} units of consumption goods at date $t + 1$, $r_{t+2}(1 - \delta)$ units at date $t + 2$, $r_{t+3}(1 - \delta)^2$ at date $t + 3$ and so on.

We make two main assumptions. First, following [Kiyotaki and Moore \(2012\)](#) we will assume that the entrepreneur with an investment opportunity faces a liquidity constraint: she can only sell a fraction $\phi_t < 1$ of her current equity holdings to raise funds for financing investment. This assumption means the entrepreneur cannot self finance and will need to borrow. Second, the entrepreneur has access to two risky technologies of producing capital, a “safer” and “riskier” technology. As we describe in the next subsection, the safer technology uses a unit of the consumption good to produce *at least* a unit of capital with a high success probability while the riskier technology uses less than a unit of the consumption good to produce *at least* a unit of capital but with a lower success probability. We further assume that investing has positive NPV only when the safer technology is used and that outsiders cannot freely observe the type of technology that has been chosen. Since the entrepreneur cannot pre-commit to use the safer technology, she must be given an incentive to do so. This “incentive compatibility” constraint together with a potential lender’s “breakeven” condition creates a credit constraint: the entrepreneur can credibly pledge only a fraction $\theta_t < 1$ of future returns.

Let a_t be the investing entrepreneur’s holding of equity (which may be of her own or others’ capital) at the end of period t . To finance investment at the beginning of the period, she can dispose of a fraction ϕ_t of her depreciated holdings from the previous period: $(1 - \delta)a_{t-1}$. She can pledge only a fraction θ_t of the gross return from investing, so at the end of the period, she must hold a fraction $(1 - \theta_t)$ of claims to the newly created capital i_t . These two “liquidity constraints” imply that $a_t \geq (1 - \theta_t)i_t + (1 - \phi_t)(1 - \delta)a_{t-1}$. In [Kiyotaki and Moore \(2012\)](#), the parameters θ_t and ϕ_t are exogenous.

In our model, the assumptions on capital production technologies create an agency problem between the entrepreneur and potential lenders. The value of θ_t will consequently depend on aspects of the corporate governance and taxation system of the economy. We develop this

relationship with the tax system in the next subsection where we introduce taxation and auditing by the tax authority after discussing the basic agency problem. The tax system enters into the agency problem because an entrepreneur who has used the riskier technology has an incentive to report the higher cost associated with the safer technology i.e. evade taxes. But auditing can reveal the true investment technology used by the entrepreneur and as such ease the borrowing constraint. In other words, the tax authority's acts as a typical "monitor" in standard models contracting with asymmetric information (e.g. [Holmstrom and Tirole, 1997](#)). Thinking of the tax authority as a monitor is not controversial and has been previously discussed in the literature ([Desai et al., 2007](#)).

2.2 The Agency Problem

Entrepreneur An entrepreneur has access to two types technologies for producing capital: safe and risky. The safe technology uses i_t units of the consumption good to produce Ri_t units of capital with probability p_H or 0 with probability $(1 - p_H)$. The risky technology uses $i_{L,t} < i_t$ units of the consumption good to produce Ri_t units of capital with probability $p_L < p_H$ or 0 with probability $(1 - p_L)$. The subscripts $\{H, L\}$ denote high and low respectively. If investment occurs at the beginning of a period t and is successful, Ri_t units of new capital are available at the end of the period. Let q_t be the price of a unit of capital at the end of date t . The gross return to investment over the period is:

$$\text{Gross Return} \equiv q_t Ri_t$$

The rate of return to investment is greater than the rate of return to capital ($R > 1 + r_t - \delta$), so the entrepreneur will always want to invest as much as possible: $i_t \in [0, \infty)$. Let a_t denote her initial assets which can either be invested or used for consumption. In order to do either, the entrepreneur needs to convert assets into consumption goods. However, since she faces the liquidity constraint, only a fraction ϕ_t of her assets can be immediately sold to raise $A_t = [r_t + q_t \phi_t (1 - \delta)] a_{t-1}$ units of the consumption-investment good. In order to invest $i_t > A_t$, the entrepreneur will need to borrow the amount:

$$\text{Borrowing} = i_t - [r_t + q_t \phi_t (1 - \delta)] a_{t-1} = i_t - A_t$$

where A_t is her "net worth" or "cash-in-hand".

Whenever investment is undertaken, the entrepreneur has private information about the technology she has used. If she chooses the safe technology, then the net return is $(q_t R - 1)i_t$. If she chooses the risky technology, then the net return is $(q_t Ri_t - i_{L,t}) = (q_t R - 1)i_t + Bi_t$, where $B = [i_t - i_{L,t}]/i_t$ and $0 < B < 1$. The private benefit $Bi_t > 0$ is not observable to any potential lender.

Lenders Both the entrepreneur and lenders (or investors) are risk neutral. Events take place within the period t so there is no time preferences and the borrower is protected by limited

liability (entrepreneur income cannot take on negative values). For now we drop the time subscript t for ease of exposition. Lenders are competitive and make zero profit. The loan contract between the lender and the borrower stipulates the following. First the contract specifies whether investment is financed and then if so, how the profits are shared between lenders and borrower. The borrower's limited liability means the in case of success, the two parties share profits Ri (the verifiable amount); R^b goes to the borrower and R^l goes to the lenders, so $R^b + R^l = Ri$. The incentive scheme for the entrepreneur is of the following form: R^b in the case of success and 0 in the case of failure. The zero profit condition for the lenders can therefore be written as:

$$p_H R^l = i - A$$

assuming the loan contract induces the borrower to chose the safe technology, then the rate of interest is defined by

$$R^l = (1 + \text{Interest})(i - A) \text{ or } (1 + \text{Interest}) = \frac{1}{p_H}$$

which reflects a default premium: $\text{Interest} = \frac{1}{p_H} - 1 > 0$ unless $p_H = 1$; i.e. the interest rate exceeds the expected zero rate of return demanded by investors.¹ We assume that with $q = 1$, the project has positive NPV per unit of investment only if the entrepreneur chooses the safe technology: $p_H Ri - i > 0$ or $p_H R > 1$ and negative NPV even when the entrepreneur's private benefit is included: $p_L Ri - i + Bi < 0$.²

The entrepreneur faces the following trade-off once financing has been obtained: by choosing the risky technology, she obtains private benefit Bi , but reduces the probability of success from p_H to p_L . Because she has a stake R^b in the firm's income, the entrepreneur will choose the safe technology if the following "incentive compatibility constraint" holds:

$$p_H R^b \geq p_L R^b + Bi \text{ or } R^b \geq \frac{Bi}{\Delta p} \quad (\text{IC}_b)$$

The highest income in the case of success that must be pledged to the lenders without jeopardizing the entrepreneur's incentive is $qRi - \frac{Bi}{\Delta p}$ and the expected *pledgeable income* is

$$\mathcal{P} \equiv p_H \left(qRi - \frac{Bi}{\Delta p} \right) = p_H (Ri - R^b)$$

and because the lender must break even in order to finance investment, a necessary condition for the borrower to receive a loan is that the expected pledgeable income exceed the borrowed amount:

$$p_H (qRi - R^b) \geq i - A \quad (\text{IR}_1)$$

which is the individual rationality, "breakeven" or "participation" constraint. The necessary

¹ In calibrating the model in Section 3, $p_H = .95$ which implies a risk premium or "interest rate" of $r_t \approx 5\%$.

² Using $p_H = .95$, $R = 1.75$ gives $p_H R = 1.66 > 1$ and $p_L = .51$, $B = .06$ gives $p_L R - 1 + B = -.0475$.

condition for financing to occur is that the entrepreneur has initial income

$$\begin{aligned} A &\geq \bar{A} = i - p_H(qRi - R^b) \\ &= \left[1 - p_H\left(qR - \frac{B}{\Delta p}\right)\right] i = (1 - \theta)i \end{aligned} \quad (2)$$

Equation (2) is the **credit constraint**, i.e: $A \geq (1 - \theta)i$. The entrepreneur's share of future returns is the fraction of investment financed from her net-worth: $\frac{A}{i} = (1 - \theta)$ and the lender's share is $\frac{i - A}{i} = \theta$. If financing occurs, then $\bar{A} \geq 0$, which implies

$$1 > p_H\left(qR - \frac{B}{\Delta p}\right) \text{ or } \theta < 1 \quad (3)$$

i.e. the NPV is smaller than the minimum expected rent that must be left to the borrower to provide her with an incentive to choose the safe technology. Condition (3) means that the borrower can lever her wealth to invest $i \leq mA$ for some multiplier

$$m = \frac{1}{1 - \theta} > 1 \quad \text{and} \quad \theta = p_H\left(qR - \frac{B}{\Delta p}\right) \quad (4)$$

The multiplier is smaller the higher the private benefit B and the lower the likelihood ratio $\frac{p_H}{\Delta p}$.

2.2.1 Taxation

Assume that the government has access to a proportional tax τ on final profit so that Profit after Tax = $(1 - \tau)(qR - 1)i = R^b + R^l$. We will introduce the possibility of misreporting profit, but for now we maintain the assumption that the entrepreneur **always** reports her income **truthfully**. The incentive compatibility constraint becomes:

$$p_H R^b \geq p_L R^b + (1 - \tau)Bi \text{ or } R^b \geq \frac{(1 - \tau)Bi}{\Delta p} \quad (IC'_b)$$

The breakeven condition for the lender to participate now becomes:

$$p_H \left[(1 - \tau)(qR - 1)i - R^b \right] \geq i - A \quad (IR'_l)$$

Again, the **credit constraint** is $A \geq (1 - \theta)i$ and the maximum investment satisfies $i \leq \bar{m}A$, where

$$\bar{m} = \frac{1}{1 - \bar{\theta}} \quad \text{and} \quad \bar{\theta} = p_H(1 - \tau) \left[(qR - 1) - \frac{B}{\Delta p} \right] \quad (5)$$

so that the multiplier is smaller the higher the tax on profit τ . This formalizes the ideas found in both the empirical and theoretical literature that corporate tax lowers investment.³ Note that the incentive compatibility condition (IC'_b) for the borrower would be equivalent to (IC_b) if we did not assume truthful reporting of income.

³See for instance [Jaimovich and Rebelo \(2017\)](#) on the role of taxes in reducing private incentives to invest. [Heider and Ljungqvist \(2015\)](#) show that an increase in taxes increases leverage: firms tend to use more debt rather than equity to finance investment.

2.2.2 Tax Evasion

We now introduce the possibility of evading tax by the entrepreneur together with audits or inspections.

Random Audits When an entrepreneur is successful, her newly produced capital is visible to anyone, so she always reports the success or failure of investment truthfully. However, if she used the riskier technology, she may choose to falsely report her profits by claiming costs associated with the safe technology. In response, the government randomly audits successful entrepreneurs to verify income. We assume that when an audit is undertaken, the tax authority or government with probability y discovers the undeclared private benefit B (evaded amount) and with probability $(1 - y)$ discovers nothing. The probability of discovery depends on the amount invested in auditing, a cost $\zeta : y = y(\zeta)$, with $y(0) = 0, y(\infty) = 1$ and $y'(\zeta) > 0$. If evasion is discovered, the entrepreneur faces a surcharge s in addition to her tax liabilities. The assumption that audits only discover evasion with probability y is without loss of generality equivalent to a model where firms are audited with the same probability and evasion is always discovered. Introducing audits in this manner makes the exposition simpler and more aligned with the corporate governance literature.

Technology Choice and Income Declaration under Auditing In the presence of audits, the entrepreneur can either declare her true income or report falsely. We give a full description of the technology and reporting choice decision facing the entrepreneur in Appendix A.1. To formalize the choice, we write the entrepreneur decision as a game where she chooses the best response to the audit (AUDIT) or no-audit (NO AUDIT) strategies of the tax authority (auditor/monitor). The entrepreneur's strategies are to play safe/truthful (SAFE) and risky/false (RISKY). We assume that when a firm is audited, it faces audit response gain or cost proportional to its level of investment ηi or $-\eta i_L$ for some small number η . This assumption is based on the findings of Guedhami and Pittman (2008) and Graham et al. (2008). In Graham et al. (2008), when a firm is forced to issue an income restatement due to fraud, it experiences a 68% increase in its loan spread: the amount the borrower pays in basis points over LIBOR (London Interbank Offered Rate) or LIBOR equivalent – across firms, income restatements increase the spread by 85 basis points (Table 2, page 49). In Guedhami and Pittman (2008) increasing the probability of an IRS audit from 19% to 33% reduces the spread by 25 basis points. In our economy, borrowing and investing are one shot per period activities, so we impute the gains or losses associated with an audit as a one time benefit to the safe or cost to the risky types. If the game was played every period, the benefit to the safe type would accrue in the next period through a lower borrowing cost. The game matrix is as follows:

Entries inside the matrix are payoffs or gains to each player when they play a given strategy, where we follow the standard convention of left entries in each cell being the row player's payoff.

		Tax Authority	
		AUDIT (y)	NO AUDIT ($1 - y$)
Firm	SAFE (x)	$p_H(R^b + \eta i)$ $-\zeta$	$p_H R^b$ 0
	RISKY ($1 - x$)	$p_L [R^b + (1 - (\tau + s))Bi - \eta i_L] + (1 - p_L)Bi$ $(\tau + s)Bi - \zeta$	$p_L R^b + Bi$ $-(\tau + s)Bi$

Figure 1: Normal Form Representation of Tech Choice with Auditing Game

In the first-row first-column entry, the entrepreneur is audited even though she is reporting true income. The audit nets her ηi while the auditor incurs the cost ζ without any gains in tax revenue. Similar arguments apply to all other entries. The second row includes the surcharge s for tax evasion when detected. Note that all entries use success probabilities p_H and p_L since only successful entrepreneurs face auditing risk (see Appendix A.1.)

To solve the game, we consider mixed strategies.⁴ Letting the entrepreneur declare her true income with probability x while auditing takes place with probability y , the expected payoff of this randomising strategy is:⁵

$$E(x, y) = x \left\{ p_H (R^b + y\eta i) \right\} + (1 - x) \left\{ p_L [R^b + Bi - y((\tau + s)B + \eta(1 - B))i] + (1 - p_L)Bi \right\} \quad (6)$$

The entrepreneur then chooses x such that any changes in y have no effect on $E(x, y)$ which occurs when $\frac{\partial E(x, y)}{\partial y} = 0$.⁶ This implies that the probability of declaring truthful income or choosing the safe technology is given by:

$$x = \frac{p_L [(\tau + s)B + \eta(1 - B)]}{p_L [(\tau + s)B + \eta(1 - B)] + p_H \eta} < 1 \quad (7)$$

which is increasing in the tax rate τ and the private benefit B so that jurisdictions characterized by high tax and poor corporate governance are *less* likely to experience tax evasion. An increase in audit costs reduces the probability of truthful reporting: $\frac{\partial x}{\partial \eta} < 0$, in line with the theoretical results of [Lipatov \(2012\)](#).

Pledgeable Income with Evasion and Random Audits We now turn our attention to pledgeable income and the investment rate under tax evasion and auditing. Let U_S^b and U_R^b be the borrowers return when choosing the safer and riskier technologies respectively. When the safe technology

⁴The game has no Nash equilibrium in pure strategies.

⁵After replacing $i_L = (1 - B)i$

⁶See Chapter 7 of [Binmore \(2007\)](#).

is chosen, the borrower always reports truthfully so $U_S^b = p_H(R^b + y\eta i)$. Similarly,

$$U_R^b = p_L [R^b + Bi - y((\tau + s)B + \eta(1 - B))i] + (1 - p_L)Bi$$

The incentive compatibility for the borrower to choose the safe technology requires that $U_S^b \geq U_R^b$ which implies that:

$$R^b \geq \frac{\{B - y(p_L[(\tau + s)B + \eta(1 - B)] + p_H\eta)\}i}{\Delta p} \equiv \frac{\omega i}{\Delta p} \leq \frac{(1 - \tau)Bi}{\Delta p} \quad (IC''_b)$$

Similar to the results in the corporate governance literature, the existence of audits reduces the private benefit enjoyed by the entrepreneur whenever $\omega \leq (1 - \tau)B$ (see e.g. [Admati, Pfleiderer and Zechner, 1994](#)). $\frac{\partial R^b}{\partial y} = -(p_L[(\tau + s)B + \eta(1 - B)] + p_H\eta) \frac{i}{\Delta p} < 0$ means that increasing the audit probability by a small amount dy , allows the lender (principal) to reduce the transfer to the entrepreneur (agent) R^b by an amount proportional to $(\tau + s)$ (see e.g. [Laffont and Martimort, 2002](#), page 125). The break even condition for the lender to participate is:

$$p_H [(1 - \tau)(qR - 1)i - R^b] \geq i - A \quad (IR''_l)$$

Again, the **credit constraint** is $A \geq (1 - \bar{\theta})i$ and the maximum investment satisfies $i \leq \bar{m}A$, where

$$\bar{m} = \frac{1}{1 - \bar{\theta}} \quad \text{and} \quad \bar{\theta} = p_H \left((1 - \tau)(qR - 1) - \frac{\omega}{\Delta p} \right) \quad (8)$$

Without auditing, $y = 0$ and nobody pays taxes on private benefits, $\tau + s = 0$ which gives $\omega = B$ and $\bar{m} > \bar{m}$.⁷ With full auditing, there is no tax evasion, $y = 1$ (everybody is audited and nobody evades: $s, \eta = 0$) and $\omega = (1 - \tau)B$ implies $\bar{m} = \bar{m}$. Since an increase in ω lowers the multiplier and hence investment; an increase in evasion which also lowers ω would decrease investment.⁸ An increase in the probability of being audited has a positive effect on the multiplier \bar{m} :

$$\frac{\partial \bar{m}}{\partial y} = \frac{p_H}{\bar{m}^2} [p_L[(\tau + s)B + \eta(1 - B)] + p_H\eta] > 0$$

so a higher probability of an audit revealing evasion increases the level of financing an entrepreneur can obtain. This result is in tandem with [Ellul, Jappelli, Pagano and Panunzi \(2016\)](#) who find that firms choose higher transparency in countries with higher audit quality and consequently enjoy better access to finance. The more interesting effect is that of a change in taxes. Defining $s = \bar{s}\tau$ for $\bar{s} > 1$, then the effect of a tax increase on the multiplier \bar{m} is

$$\frac{\partial \bar{m}}{\partial \tau} = \frac{p_H}{\bar{m}^2} [-(qR - 1) + yp_L B(1 + \bar{s})]$$

⁷With $\omega = B$, we have $\bar{\theta} = p_H \left((1 - \tau)(qR - 1) - \frac{B}{\Delta p} \right) < p_H \left((1 - \tau)(qR - 1) - (1 - \tau) \frac{B}{\Delta p} \right) = \bar{\theta}$. This happens because we assumed truthful reporting when introducing taxation. Naturally, if entrepreneurs can artificially inflate costs to reduce the tax burden without consequence, returns are higher and so is investment.

⁸In a fixed investment model, a lower \bar{m} would increase the initial wealth required by an entrepreneur to make an investment.

whose sign depends on the price of equity q .

2.3 Macroeconomic Equilibrium

Since our model is very close to that of [Kiyotaki and Moore \(2012\)](#), the exposition henceforth will closely follow their work. We omit some details and refer the interested reader to the original paper. The timing of events in the economy is as follows:

-
1. Aggregate productivity χ_t is realized and production takes place.
 2. π is revealed. Investing agents choose consumption, sell a fraction ϕ_t of their depreciated asset holdings. Non-investing agents choose consumption and purchase assets from investing agents.
 3. Within period capital production occurs subject to moral hazard.
 4. $[xp_H + (1-x)p_L]Ri_t$ units of new capital are added to the economy at the end of the period.
-

From section 2.1, recalling that c_t is consumption, a_t is the equity holding, priced at q_t , and that the return to equity equals that of capital r_t , the investing entrepreneur's flow of funds constraint is:

$$c_t^b + i_t = [(1-\tau)r_t + \tau\delta q_t + (1-\delta)\phi_t q_t]a_t^b + (1-\tau)r_{z,t}z_t^b + \theta_t i_t \quad (9)$$

where the superscript b on variables stands for borrower. This equation says that in order to finance consumption c_t and investment i_t , the entrepreneur issues equity $\theta_t i_t$ priced at unity together with the maximum after tax liquidity obtained from dividends $[(1-\tau)r_t + \tau\delta q_t]a_{t-1}^b$, liquid assets $(1-\tau)r_{z,t}z_{t-1}^b$ and the resalable fraction of depreciated equity $(1-\delta)\phi_t q_t a_t^b$. In equation (9), $\tau\delta q_t$ is depreciation allowance and z_{t-1} represents assets that are fully liquid; a form of storage with return $r_{z,t} = (1 - (z_t/z_0)^{\bar{z}})$, $z_0, \bar{z} > 0$. The investing entrepreneur's end of period asset holding is:

$$a_{t+1}^b = (1-\theta_t)[xp_H + (1-x)p_L]Ri_t + (1-\phi_t)(1-\delta)a_t^b \quad (10)$$

which is her retained fraction $(1-\theta_t)$ of the newly produced capital $[xp_H + (1-x)p_L]Ri_t$ plus the unsold fraction of her depreciated initial asset holding $(1-\phi_t)(1-\delta)a_t^b$. Solving this equation forward for investment:

$$i_t = \frac{a_{t+1}^b - (1-\phi_t)(1-\delta)a_t^b}{(1-\theta_t)[xp_H + (1-x)p_L]R}$$

and substituting into (9) gives:

$$c_t^b + q_t^R a_{t+1}^b = \left[(1-\tau)r_t + \tau\delta q_t + (1-\delta) \left(\phi_t q_t + (1-\phi_t)q_t^R \right) \right] a_t^b + (1-\tau)r_{z,t}z_t^b \quad (11)$$

where $q_t^R = \frac{1}{[xp_H + (1-x)p_L]R}$ is the effective replacement cost of equity for the investing entrepreneur: she has to make a downpayment $(1 - \theta_t)$ for each unit of investment for which she retains a fraction $(1 - \theta_t)[xp_H + (1-x)p_L]R$, so she needs q_t^R to retain a unit claim to the capital she has produced. The RHS of the equation is her net worth which is gross-dividend from equity and storage plus the value of her depreciated equity $(1 - \delta)a_t^b$ of which the resalable fraction ϕ_t is valued at the market price q_t and the non-resaleable fraction $(1 - \phi_t)$ is valued at the effective replacement cost q_t^R . Using (9) we obtain investment as:

$$i_t = \frac{[(1 - \tau)r_t + \tau\delta q_t + (1 - \delta)\phi_t q_t]a_t^b + (1 - \tau)r_{z,t}z_{t-1}^b - c_t^b}{(1 - \theta_t)} \quad (12)$$

which simply says that i_t equals the ratio of liquidity available after consumption to the required downpayment for investment. Next consider the entrepreneur who does not have an investment opportunity (lender) and in line with previous notation, let the superscript l tag her variables. Her flow of funds constraint is

$$c_t^l + q_t a_{t+1}^l = [(1 - \tau)r_t + \tau\delta q_t + q_t(1 - \delta)]a_t^l + (1 - \tau)r_{z,t}z_{t-1}^l$$

The LHS is her purchase of consumption and new equity holdings and the RHS is her income from dividends and storage plus the market value of her depreciated equity holdings, assuming the resaleability constraint does not hold. We now determine the optimality conditions. Let the superscript i, j = b, l tag variables for an agent who is of type i in period $t - 1$ and type j in period t . For instance, date t consumption of an agent who was a borrower in the previous period and is currently a lender is denoted by: c_t^{bl} . The optimality conditions with respect to a_{t+1}^l, a_{t+1}^b for agents of type ij together with the trade-off between holding equity and storage, are given by: (see A.2 in the Appendix):

$$\frac{q_t}{c_t^{ll}} = \beta\pi E_t \left\{ \frac{(1 - \tau)r_{t+1} + \tau\delta q_{t+1} + (1 - \delta)(\phi_{t+1}q_{t+1} + (1 - \phi_{t+1})q_{t+1}^R)}{c_{t+1}^{lb}} \right\} \quad (13a)$$

$$+ \beta(1 - \pi)E_t \left\{ \frac{(1 - \tau)r_{t+1} + \tau\delta q_{t+1} + (1 - \delta)q_{t+1}}{c_{t+1}^{ll}} \right\}$$

$$\frac{q_t^R}{c_t^{lb}} = \beta\pi E_t \left\{ \frac{(1 - \tau)r_{t+1} + \tau\delta q_{t+1} + (1 - \delta)(\phi_{t+1}q_{t+1} + (1 - \phi_{t+1})q_{t+1}^R)}{c_{t+1}^{bb}} \right\} \quad (13b)$$

$$+ \beta(1 - \pi)E_t \left\{ \frac{(1 - \tau)r_{t+1} + \tau\delta q_{t+1} + (1 - \delta)q_{t+1}}{c_{t+1}^{bl}} \right\}$$

$$c_t^{ll} = c_t^{bl} \quad (13c)$$

$$c_t^{bb} = c_t^{lb} \quad (13d)$$

$$\frac{(z_t/z_0)^{\bar{z}}}{c_t^{ll}} = \beta E_t \left\{ \pi \frac{1}{c_{t+1}^{lb}} + (1 - \pi) \frac{1}{c_{t+1}^{ll}} \right\} \quad (13e)$$

We now consider the aggregate economy. The linearity of consumption, investment and savings choices means aggregation can be done without the need to keep track of distributions. Aggregate holdings of equity equals the aggregate capital stock K_{t-1} . At the start of date t , a fraction π of K_{t-1} is held by entrepreneurs who have an investment opportunity. Letting Z_t denote aggregate storage, then from (12), total investment I_t in new capital satisfies:

$$\begin{aligned} I_t = i_t^{bb} + i_t^{lb} &= \frac{[(1-\tau)r_t + \tau\delta q_t + (1-\delta)\phi_t q_t] (a_t^{bb} + a_t^{lb}) + (1-\tau)r_{z,t}(z_{t-1}^{bb} + z_{t-1}^{lb}) - c_t^{bb} - c_t^{lb}}{(1-\theta_t)} \\ &= \frac{[(1-\tau)r_t + \tau\delta q_t + (1-\delta)\phi_t q_t] \pi K_{t-1} + (1-\pi)\pi Z_{t-1} - c_t^{bb} - c_t^{lb}}{(1-\theta_t)} \end{aligned} \quad (14)$$

where the last equality has used (i) $a_t^{bb} = \pi a_{t-1}^b = \pi^2 K_{t-1}$, $a_t^{lb} = \pi a_{t-1}^l = \pi(1-\pi)K_{t-1}$ and $a_t = K_t$ and (ii) $z_{t-1}^{bb} = 0$, $z_t^{lb} = (1-\pi)\pi Z_t$, since agents who had an investment opportunity in the previous period do not accumulate storage. Goods market clearing requires total output $Y_t = \chi_t^\alpha K_{t-1}^\alpha$ ⁹ plus storage brought forward Z_{t-1} to be equal to investment I_t plus consumption $C_t = c_t^{bb} + c_t^{bl} + c_t^{lb} + c_t^{ll}$, new storage $(z_t/z_0)^{\bar{z}} Z_t$ and tax revenue T_t :

$$Y_t + Z_{t-1} = I_t + C_t + (z_t/z_0)^{\bar{z}} Z_t + T_t \quad (15)$$

where the tax revenue comprises of the tax from returns to capital and storage plus the tax on profit from investment of truthful and untruthful reporters. The untruthful reporters consist of two groups: audited and unaudited.

$$\begin{aligned} T_t &= \tau(r_t - \delta q_t)K_{t-1} + \tau r_{z,t} Z_{t-1} + x[p_H \tau(q_t R - 1)]I_t \\ &\quad + (1-x)[(1-y)\{p_L \tau(q_t R - 1)\} + y\{p_L \tau(q_t R - 1) + (\tau + s)B\}]I_t \end{aligned} \quad (16)$$

Finally, investing entrepreneurs sell a fraction θ_t of claims to the outcome of their investment $[xp_H + (1-x)p_L]RI_t$ together with a fraction ϕ_t of their depreciated equity holdings $\pi(1-\delta)K_{t-1}$. The stock of equity held by the non-investing entrepreneurs at the beginning of period $t+1$ is therefore

$$\theta_t[xp_H + (1-x)p_L]RI_t + \phi_t \pi(1-\delta)K_{t-1} + (1-\delta)(1-\pi)K_{t-1} \equiv A_{t+1}^1$$

The stock held by investing entrepreneurs is their retained equity from investment outcome $(1-\theta_t)[xp_H + (1-x)p_L]RI_t$ plus their unsold depreciated equity holdings $(1-\phi_t)\pi(1-\delta)K_{t-1}$:

$$(1-\theta_t)[xp_H + (1-x)p_L]RI_t + (1-\phi_t)(1-\delta)\pi K_{t-1} \equiv A_{t+1}^b$$

⁹From equation (1), $Y_t = \chi_t^{\frac{\alpha}{\varphi}} K_{t-1}^\alpha$. Since the all $k_{j,t-1}$ are perfect substitutes, the elasticity of substitution $\sigma = \frac{1}{1-\varphi} \rightarrow \infty$, which implies $\varphi \leftarrow 1$, so aggregate output equals $Y_t = \chi_t^\alpha K_{t-1}^\alpha$

The aggregate capital stock therefore evolves according to:

$$K_t = A_{t+1}^l + A_{t+1}^b = [xp_H + (1-x)p_L]R_t + (1-\delta)K_{t-1} \quad (17)$$

Define the prices q_t^R , r_t and the fraction of pledgeable future returns θ_t as:

$$q_t^R = \frac{1}{[xp_H + (1-x)p_L]R} \quad (18)$$

$$r_t = \alpha \chi_t^\alpha K_{t-1}^{\alpha-1} \quad (19)$$

$$\theta_t = p_H \left[(1-\tau)(q_t^R R - 1) - \frac{\omega}{\Delta p} \right] \quad (20)$$

$$\omega = B - y(p_L[(\tau+s)B + \eta(1-B)] + p_H\eta) \quad (21)$$

and the evolution of aggregate productivity χ_t and liquidity ϕ_t as

$$\chi_t = (1-\rho_z)\bar{\chi} + \rho_z\chi_{t-1} + e_{\chi,t} \quad (22)$$

$$\phi_t = (1-\rho_\phi)\bar{\phi} + \rho_\phi\phi_{t-1} + e_{\phi,t} \quad (23)$$

We can now define the equilibrium. A recursive competitive equilibrium is a function $(C_t, I_t, T_t, K_t, Z_t)$ of the aggregate state $(K_{t-1}, Z_{t-1}, \chi_t, \phi_t)$ with prices (q_t^R, q_t, r_t) and credit constraint θ_t , that satisfies equations (13)–(23).

3 Quantitative Analysis

In this section, we explore the model's quantitative predictions by calibrating and solving it numerically using perturbation methods. We employ the Dynare suite of programs.

3.1 Calibration

We calibrate the model following current standards the macroeconomic literature. We divide parameters into two categories and fix the time period to a quarter. Table 1 lists parameters related to the agency problem and taxation system. The values in Table 1 are based on various estimates found in the literature. The value of $B = 0.06$ comes from estimates by Dyck and Zingales (2004, Table XI: Private Benefits of Control and Legal Origin) for English origin legal jurisdictions. The probability of success for an investment project using the safer technology is $p_H = 0.95$, which is calibrated to match a risk premium of 5%¹⁰ while $p_L = 0.51$ is chosen such that the risky technology has a slightly better than fair chance of success. $\tau = 35\%$ is based on estimates of the effective corporate tax rate on investment from the Tax Foundation Special Report 2014.¹¹ The surcharge s on misreporting costs, is based on IRS Code Section 6662 which includes the Accuracy Related Penalty of 20% of total understatement of tax or 40% for Gross Valuation Misstatements. We choose the latter value and calibrate $s = 1.4 \times \tau$.

¹⁰See footnote 1 in Section 2.2.

¹¹<https://files.taxfoundation.org/legacy/docs/SR214.pdf>

The probability of an audit y is calibrated based on Table 9a of the United States Internal Revenue Service Databook.¹² For the period October 1, 2015 to September 30, 2016, the audit rate is $y = 9.5\%$.¹³ For earlier periods, such as the fiscal year 2014, $y = 12.2\%$ which we use here. The audit gain/cost parameter is calibrated as $\eta = 1\%$ of project value. This is based on estimates from Guedhami and Pittman (2008) who approximate that a firm experiences a 25 basis point decrease in its cost of credit when its IRS audit probability increases from 19% to 35%. In our case, only successful firms are audited, so the probability of a safe firm being audited $y \times p_H = 0.12 \times 0.95 \approx 11\%$ and that of a risky firm is: $y \times p_L = 0.12 \times 0.51 \approx 6\%$. Multiplying these values by 3, gives the approximate change in audit probability the required for a firm to benefit from lower credit cost, i.e. $3 \times (11\%, 6\%) \approx (35\%, 19\%)$, so we multiply the 25 basis points by 3 and $\eta = 0.0075 \approx 1\%$. For these parameter values, $x = 0.76$ and $\omega = 0.0258$.

Table 1: Calibration of model parameters for x

Parameter	Description	Value
B	Private Benefit	0.06
η	Audit Response Cost	0.01
p_H	Safety Tech Success Prob.	0.95
p_L	Risky Tech Success Prob.	0.51
τ	Corporate Tax Rate	0.35
s	Surcharge for evasion	$1.4 \times \tau$
y	Audit Prob.	0.12

Figure 2 shows how changes in the parameters related to auditing, tax rates and governance affect the level of evasion x . The top panel shows a smooth fall in x as the cost of an audit increases, in line with the results of Lipatov (2012). The middle and bottom panels show a smooth increase as the tax rate (τ) and the private benefits (B), respectively increase. The intuition for these results is straightforward: a higher tax rate or private benefit lowers the amount of funds the entrepreneur can raise from outsiders so she has a higher stake in the outcome of investment which raises her preference for the safe technology.

The effects of tax changes tends to vary with the quality of corporate governance. To see this, we solve for x over a range covering the estimates of private benefits of control from Dyck and Zingales (2004). In their Table XI, they give estimates of B based on the legal origin of different jurisdictions. These are Scandinavian, English, German, French and Soviet, for which the private benefits are respectively, $B = 0.048, 0.055, 0.109, 0.212$ and $B = 0.356$. We plot the values of x for different combinations B and τ while holding all other parameters as in Table 1. The results are shown in Figure 3 below. Low taxes with moderately strong governance institutions result into a low x .

Table 2 lists parameters related to the macroeconomic model and their values. Most of our parameters are set to follow the quantitative analysis of the Kiyotaki and Moore model by Bigio and Schneider (2017) and Del Negro, Eggertsson, Ferrero and Kiyotaki (2017). We

¹²See e.g. <https://www.irs.gov/pub/irs-soi/16databk.pdf>

¹³Table 9a, column (3) of Returns Examined, Large Corporations

Figure 2: How changes in audit cost, taxation and governance affect technology choice

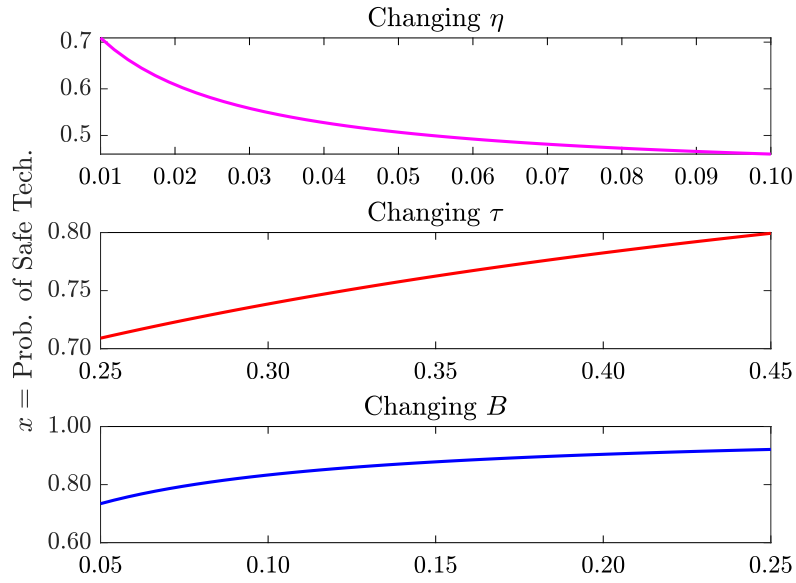
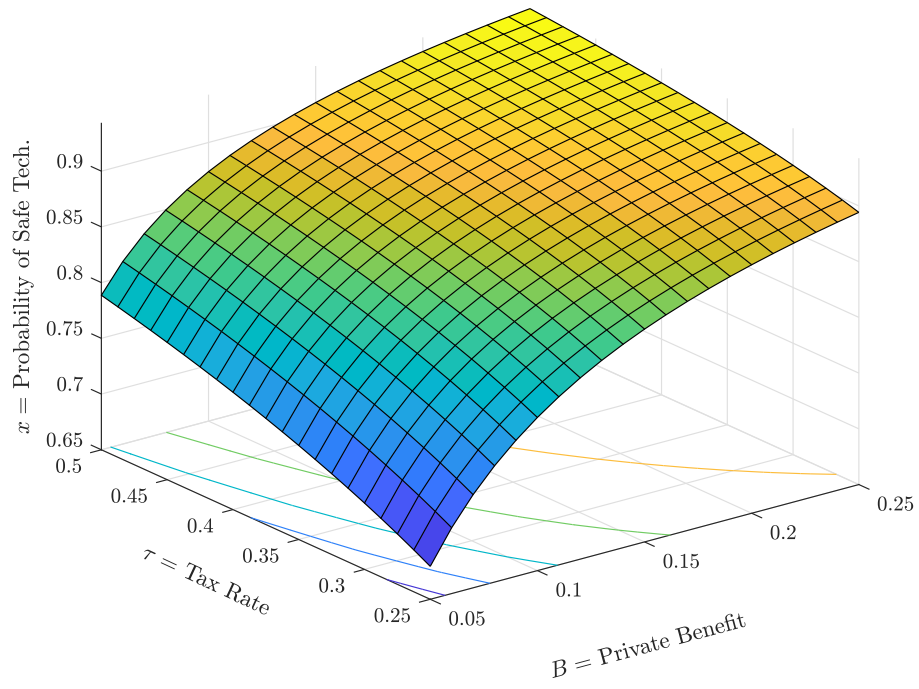


Figure 3: How changes in taxation and governance affect technology choice



set the discount factor $\beta = 0.99$, the capital share to $\alpha = 0.36$ and $\delta = 0.025$ to match the steady state investment to capital ratio $\frac{I}{K} \approx \delta$. These numbers are well established in the RBC literature. We set our value of R , the gross-return to investment to 1.77, so that our value of effective equity replacement cost matches that of [Bigio and Schneider \(2017\)](#). Following the same authors, we set the arrival of investment opportunities to $\pi = 0.012$ and the persistence of shocks to productivity and resaleability $\rho_z = \rho_\phi = 0.95$. The steady state resaleability value is set at the maximum value of $\bar{\phi} = 0.375$ which is slightly higher than the number used in the literature so far which is about 0.3. We require the slightly higher value in order to obtain reasonable values of θ and q . We set the aggregate productivity level at $\bar{\chi} = 0.35 \approx \alpha$, because our model has capital as the only factor of production, so that aggregate output equals the return to holding capital. This choice also ensures that our steady state value of the capital stock is close to the theoretical moments found in the literature. The parameter values in Table

Table 2: Baseline Calibration: Macroeconomic Model Parameters

Parameter	Description	Value
α	Capital Share	0.36
β	Discount factor	0.99
δ	Depreciation Rate	0.025
\bar{z}, z_0	Storage Parameters	0.1, 0.5
π	Arrival of investment opportunity	0.012
R	Gross Return to Capital Investment	1.77
$\bar{\chi}$	Productivity	0.35
$\bar{\phi}$	Resaleable fraction of equity	0.375
ρ_χ, ρ_ϕ	Persistence of shocks	0.95
x	Truthful Report Probability	0.76
ω	Private Benefit Multiplier	0.026

1 imply a probability of truthful income reporting by an investing entrepreneur is $x = 0.76$ which reduces the private to $\omega = 0.026$. These are the last two entries in Table 2.

3.2 Steady State

In the literature, the mortgageable fraction of new investment θ_t is usually calibrated as a model parameter. In our case, the value of θ is determined in equilibrium as it depends on the market value of capital q_t . As noted by [Kiyotaki and Moore \(2012\)](#) and [Bigio and Schneider \(2017\)](#), the value of θ sets the upper bound for the spot price of equity: $q_t \leq 1/\theta_t$ which our equilibrium outcome should satisfy. Furthermore, the equilibrium outcome should also satisfy $0 \leq \theta_t \leq 1 - \pi = 0.998$. Our basic calibration and the ensuing model solution satisfies these conditions as shown in Table 3. Our basic calibration gives a value of $\theta_t = 0.81 \leq 1 - \pi = 0.998$ which implies an upper bound of $q_t \leq 1.24$ which our equilibrium equity price of $q_t = 1.11$ satisfies. Our equilibrium value of θ matches the calibrations of [Bigio and Schneider \(2017\)](#) and [Del Negro et al. \(2017\)](#) who use the values $\theta = 0.77, 0.79$ respectively. Our market value of equity is very close to that obtained by [Bigio and Schneider \(2017, Table 4, Theoretical Moments\)](#) which is $q = 1.09$ and our choice of $R = 1.77$ implies that our effective equity

Table 3: Steady State

Variable	Description	Value
θ_t	Mortgageable fraction of equity	0.81
q_t	Market value of equity	1.11
q_t^R	Effective equity replacement cost	0.66
C_t	Consumption	1.89
I_t	Investment	0.63
K_t	Capital	37.99
Y_t	Output	2.54
Z_t	Storage	0.63

replacement cost, $q_t^R = 0.67$ exactly matches the value implied by their equilibrium where $q_t^R = \frac{1-\theta q_t}{1-\theta} = 0.67$. Our equilibrium also fits the values of the investment-output ratios targets for macroeconomic models: $\frac{I_t}{Y_t} = 0.25$ which is equivalent to the equilibrium target matched by [Del Negro et al. \(2017\)](#). In our case, the steady state investment to capital ratio is given by $\frac{I}{K} = \frac{\delta}{[xp_H + (1-x)p_L]R} = 0.0326$ but our equilibrium investment-capital ratio = 1.66% which is slightly higher than the approximately 1.23% quarterly estimates of capital-growth matched by [Perez-Orive \(2016\)](#).

In our economy changes in asset prices and macroeconomic variables are dependent on parameters that are subject to policy changes such as the tax rate τ , the audit rate y and/or regulations that affect the quality of corporate governance B . These have direct and indirect effects on the equilibrium asset prices. We now evaluate the effect of changes in some of these policy parameters on equilibrium outcomes.

3.2.1 Tax Changes

To evaluate the effects of tax changes, we perform a deterministic simulation where the tax rate τ enters our model as an exogenous variable. We let all parameters in our model economy at the values in [Table 2](#) while changing the tax rate τ in two directions, each by 7.5 percentage points. We use this value as it is close to the tax rate at which x bottoms out for $B = 0.06$ in [Figure 3](#) for a tax decrease. In the first case, the tax rate falls from $\bar{\tau} = 35\%$ to 27.5% and in the second case, it rises to 42.5%. These changes have effects on both the tech choice parameter x and the multiplier ω . When the tax rate decreases to $\tau = 27.5\%$, there is a fall in truthful reporting $x = 0.73$ while the private benefits slightly rises to $\omega = 0.027$. This raises the equilibrium equity price to $q_t = 1.13$ which also raises the mortgageable fraction of new capital to $\theta_t = 0.85$. While the mortgageable fraction of equity is higher, the overall output, consumption and investment is lower with low taxes. In the opposite direction, when the tax rate increases to $\tau = 42.5\%$, then $x = 0.79$ and $\omega = 0.025$. The equilibrium equity price is $q_t = 1.11$, the same as in the baseline steady state. The equilibrium mortgageable fraction of new capital slightly rises to $\theta_t = 0.82$. We summarize these effects in [Figure 4](#) where we trace the path of transition from the initial steady state defined by the baseline calibrations in [Tables 1 & 2](#). The simulation assumes a perfect foresight economy where there is a permanent change in τ occurring after 16

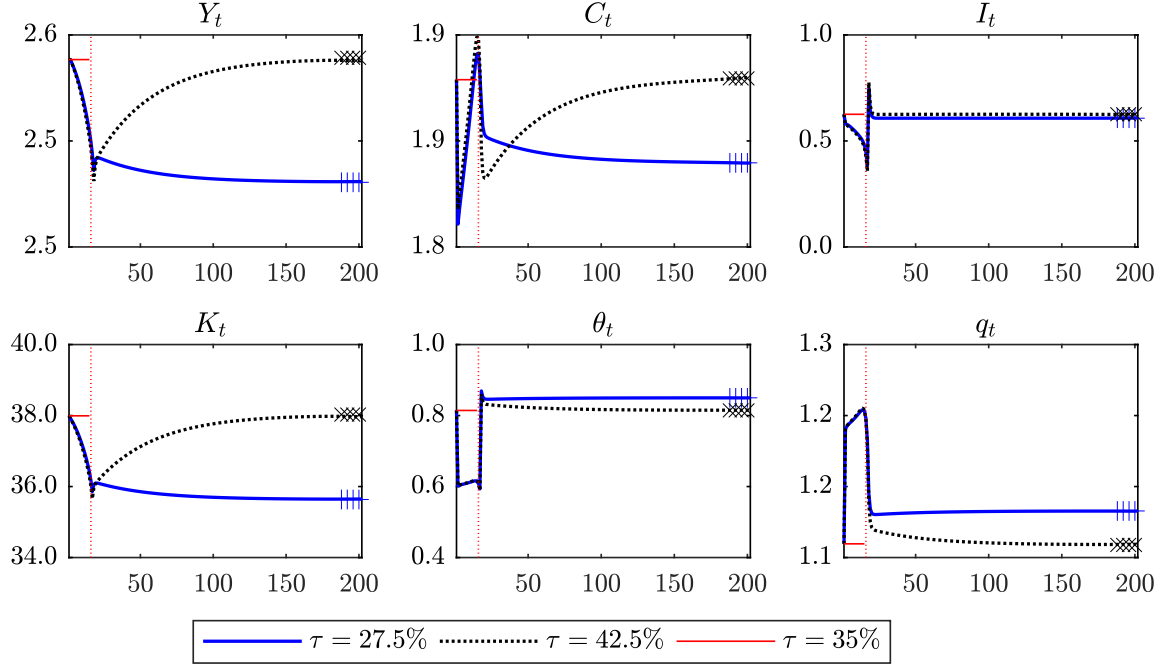


Figure 4: Effects of Tax Changes in Deterministic Equilibrium

Note: The figure compares the response of output (Y_t), consumption (C_t), investment (I_t), capital (K_t), mortgageable fraction of new capital (θ_t) and market value of equity (q_t) to a decrease (solid line) and an increase (dotted line) in the tax rate. The simulation starts at the baseline calibration (short horizontal line) with tax changes occurring after 16 periods (vertical line). Final steady states are marked by +s (decrease) and xs (increase).

periods (quarters). A decrease in taxes lowers output, consumption, capital while raising asset prices. The higher asset prices consequently increase the mortgageable fraction of new capital θ_t . There is a simple explanation for this result. Lower taxes have two effects. First, a low tax reduces the required threshold of wealth for borrowing, so θ_t and the investment multiplier \bar{m} increases. Second, a lower tax reduces the penalty of evasion from $s = 1.4 \times \tau = 49\%$ to $s = 35\%$ so the incomes of entrepreneurs is higher. However, the low evasion penalty implies that more entrepreneurs are using the risky technology, which has the overall effect of *reducing* investment, output and consumption.

3.2.2 Corporate Governance

One of the policies that can have an impact on our equilibrium is a change in the quality of corporate governance. Such changes can occur for instance when there are changes in financial regulations that affect the value of B . As an example of how regulations are related to the parameter B , suppose capital in our economy represents housing and equity was consequently a claim to the stream of payments a buyer (mortgage holder) promised to make upon purchase. In this scenario, the producer of new capital goods is a bank that bundles together housing loans that promise to pay dividends every time period in the form of mortgage repayments, similar to the “securitization” of loans that preceded the US sub-prime crisis.

We can think of the safe technology as a bank that does its due diligence by incurring the

costs of screening potential borrowers. This cost lowers B but increases the chance success $p = p_H$, i.e. a borrower that will make repayments. However, the bank can also spend less in screening and originate loans with a high likelihood of default, which is cheaper so B is high and $p = p_L$. A loosening of financial regulations would allow banks to divert more resources (high B) without consequence and seek to benefit from such behaviour by mimicking the returns of a bank with high B , that is, engage in tax evasion. In our model, the effect of this policy/regulatory change is simply captured by changes in B . We perform the change by raising and lowering B by 0.02. The lower value is similar to moving to a scenario with Scandinavian origin legal jurisdiction described in Figure 3. The effects of these changes on evasion are $x = 0.69$, $\omega = 0.017$

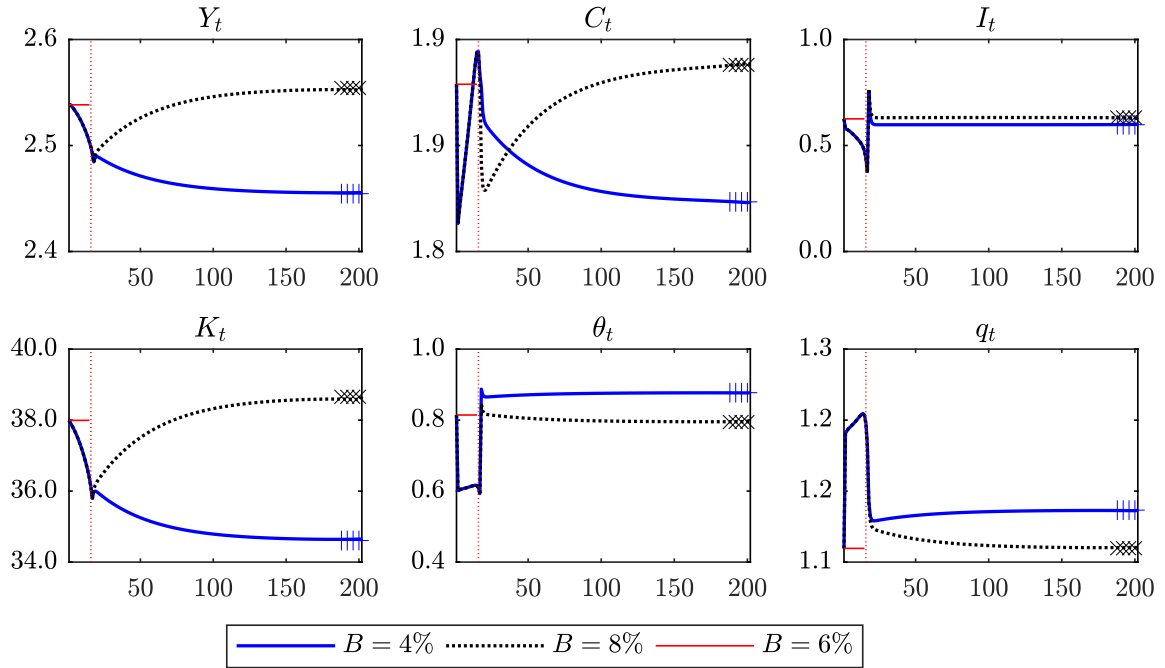


Figure 5: Effects of Private Benefit Changes in Deterministic Equilibrium

Note: The figure compares the response of output (Y_t), consumption (C_t), investment (I_t), capital (K_t), mortgageable fraction of new capital (θ_t) and market value of equity (q_t) to a decrease (solid line) and an increase (dotted line) in the private benefit B . The simulation starts at the baseline calibration (short horizontal line) with the changes occurring after 16 periods (vertical line). Final steady states are marked by +s (decrease) and x's (increase).

when $B = 0.04$ and $x = 0.80$, $\omega = 0.035$ when $B = 0.08$. The results on equilibrium are as summarized in Figure 5. A low B raises θ_t directly through ω which is lower and indirectly through the rise in q_t as a result of the fall in investment and the capital stock. The latter two effects are a result of using the riskier capital production technology by entrepreneurs who have a lower stake in the outcome of investment. A higher B lowers the mortgageable fraction of new capital θ_t , raising x , capital, output and consumption. A summary of the effects of policy changes on macroeconomic variables is given in Table 4.

Table 4: Effects of Policy Changes on Equilibrium

Variable	Tax Rates & Governance Values					
	$\tau = 35\%$ B = 6%	$\tau = 27.5\%$ B = 6%	$\tau = 42.5\%$ B = 6%	$\tau = 35\%$ B = 4%	$\tau = 35\%$ B = 8%	$\tau = 42.5\%$ B = 8%
Y_t	2.538	2.48	2.539	2.455	2.554	2.577
C_t	1.879	1.84	1.88	1.823	1.888	1.903
I_t	0.626	0.607	0.626	0.598	0.632	0.635
K_t	37.993	35.634	38.028	34.608	38.657	39.619
Z_t	0.633	0.626	0.633	0.621	0.637	0.639
θ_t	0.814	0.85	0.815	0.877	0.795	0.782
q_t	1.11	1.133	1.109	1.137	1.11	1.10
x	0.762	0.725	0.791	0.699	0.804	0.830
w	0.026	0.027	0.025	0.017	0.035	0.034

3.3 Productivity and Liquidity Shocks

We evaluate the effects of a negative productivity and liquidity shocks in our economy. Figures 6 and 7 show the impulse response functions to a 1% and a 10% decrease in χ_t and ϕ_t respectively, under three scenarios: baseline calibration, high and low taxes.

Because capital is predetermined and $\bar{\chi} = 0.35$, the 1% shock in χ_t decreases output by $0.35^\alpha - (0.35 - 0.035)^\alpha \approx 3.7\%$. Then from the goods market clearing condition, asset prices have to fall in line with productivity in order to reduce consumption and investment in line with the lower output. Investment is more sensitive to asset prices and falls more than output, given that the mortgageable fraction of equity is also falling as a result of lower asset prices.

When a liquidity shock occurs, the investing entrepreneurs are less able to finance down payment from selling their equity holdings, so investment falls substantially. Given constant depreciation, capital and output gradually fall with persistently lower investment. Lower investment means consumption must rise to maintain the goods market equilibrium. This occurs through a wealth effect in rising equity prices, which also increase the value of θ_t . Similar mechanisms are observed in the case with different values of B displayed in Figures 8 and 9. The fall in investment following a liquidity shock is larger with low taxes and the gradual falls in output and capital also larger.

4 Conclusion

This paper is part of a recent literature on macroeconomics with financial frictions, that includes the seminal works of Gertler and Kiyotaki (2010) and recently Del Negro, Eggertsson, Ferrero and Kiyotaki (2017). The common theme of this literature has been some kind of credit constraint, which has always been rationalized on the basis of asymmetric information between lenders and borrowers. In most of this literature, the borrowing constraint is generally specified in a reduced form manner.

Building on the seminal work of Holmstrom and Tirole (1997), and the prominent role of

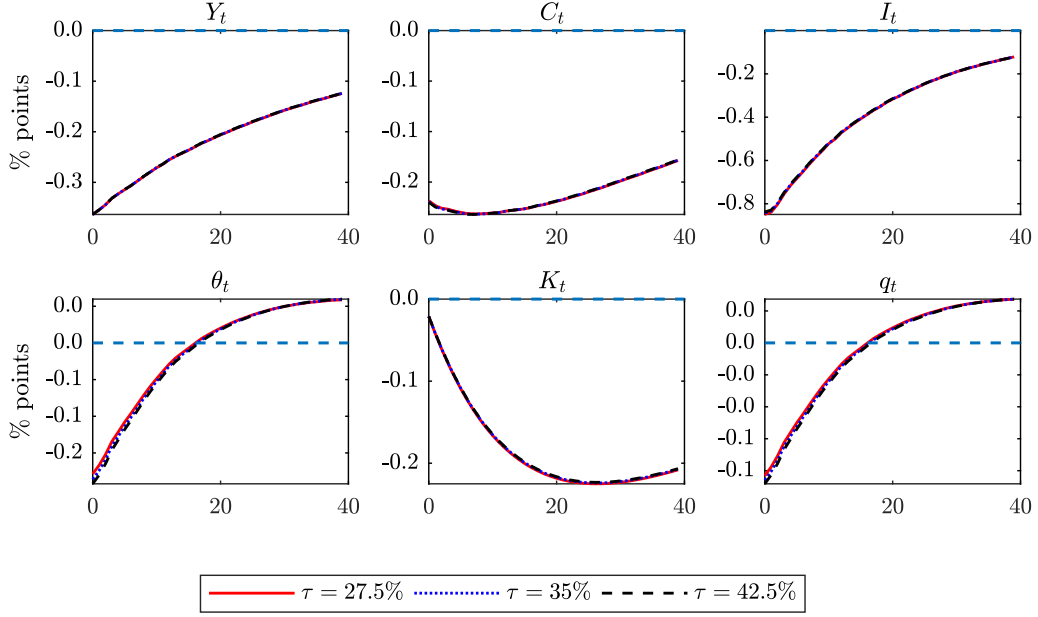


Figure 6: Impulse response a 1% negative shock in productivity χ_t under different tax regimes

Note: The figure compares the impulse response of output (Y_t), consumption (C_t), investment (I_t), mortgageable fraction of new capital (θ_t), capital (K_t) and the market price of equity (q_t) to a 1% shock to the productivity process χ_t in baseline calibration and with high and low taxes. The simulations start at the respective steady states.

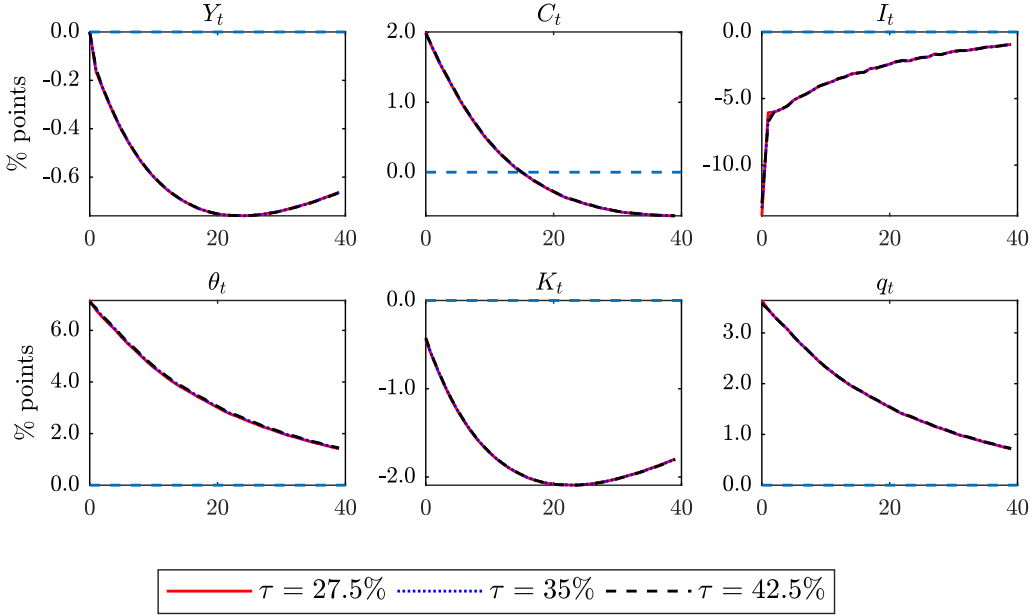


Figure 7: Impulse response a 10% negative shock in resaleability of equity ϕ_t under different tax regimes

Note: The figure compares the impulse response of output (Y_t), consumption (C_t), investment (I_t), mortgageable fraction of new capital (θ_t), capital (K_t) and the market price of equity (q_t) to a 10% shock to the process ϕ_t in baseline calibration and with different tax rates. The simulations start at the respective steady states.

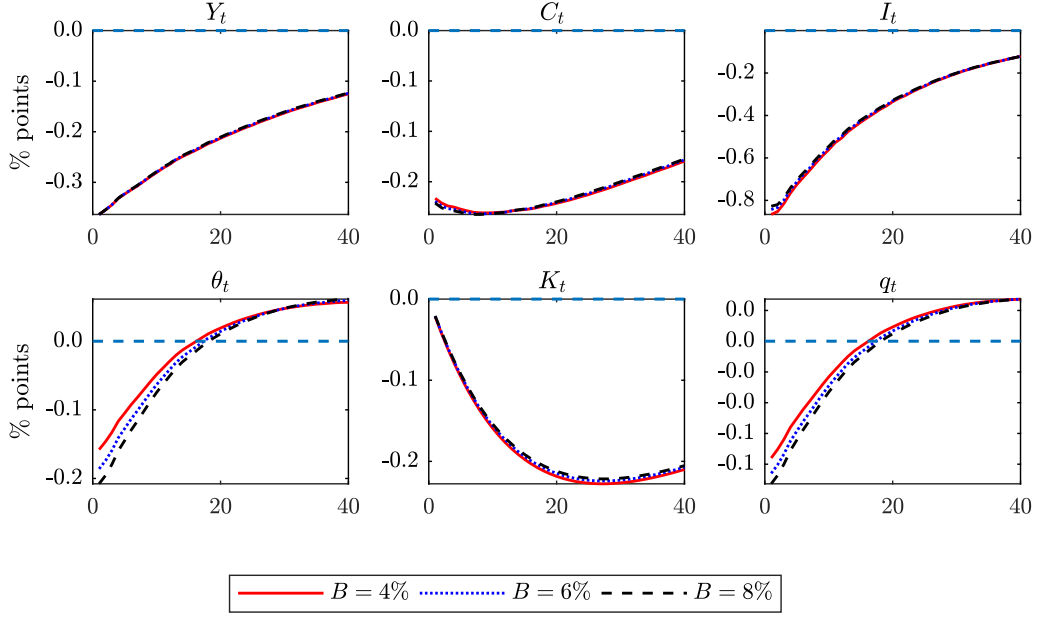


Figure 8: Impulse response a 1% negative shock in productivity χ_t under different Corporate Governance regimes

Note: The figure compares the impulse response of output (Y_t), consumption (C_t), investment (I_t), mortgageable fraction of new capital (θ_t), capital (K_t) and the market price of equity (q_t) to a 1% S.D. shock to the productivity process χ_t with different values of B . Simulations start at the respective steady states.

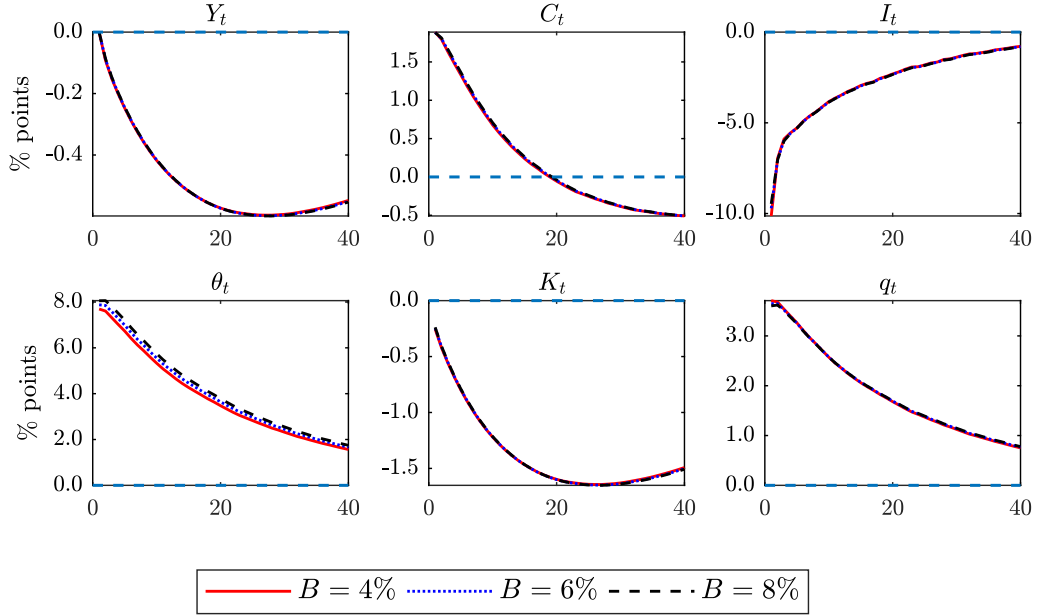


Figure 9: Impulse response a 10% negative shock in resaleability of equity ϕ_t under different Corporate Governance regimes

Note: The figure compares the impulse response of output (Y_t), consumption (C_t), investment (I_t), mortgageable fraction of new capital (θ_t), capital (K_t) and the market price of equity (q_t) to a 10% shock to the process ϕ_t under different B values.

taxation and auditing in business decisions, we have developed a microfounded economic environment in which the size of the borrowing constraint arises as an equilibrium outcome rather than a parameter calibrated from the data. While we have used the tax system and invoked the existence of evasion to motivate our moral hazard problem, our model still replicates many standard features of macroeconomic models with financial frictions. An important contribution of our work is in providing a link between aspects of corporate governance and macroeconomic outcomes.

Our goal was to show that the tax system in interaction with the quality of corporate governance, has an effect on the impact of productivity and liquidity shocks. While we do not find any large effects, our results suggest that allowing controlling shareholders to have a larger stake in investment outcomes reduces excessive risk taking. High taxes and moderate governance institutions can achieve such goals.

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A Appendix

A.1 Technology Choice Game

Figure A1 gives an extensive form representation of the technology choice and profit reporting scheme for an entrepreneur/firm. The firm moves first, choosing either the safe (with probability x) or risky (with probability $1 - x$) technology. Once this done, “nature” moves, deciding if the firm succeeds or fails with probabilities following the firm’s technology choice. The safe firm either fails or succeeds and always reports its income truthfully and then gets audited or not. The risky firm gets to move again once the outcome of investment is realized. It can issue a true or false report, after which the auditor moves to audit or not. Entries at the end represent payoffs to each player. As discussed in the main text, the entrepreneur gets R^b if she succeeds and zero otherwise. Following the findings of Guedhami and Pittman (2008) and Graham et al. (2008), the entrepreneur either benefits or losses from the outcome of an audit.¹⁴ If she is audited, she gains ηi if she used the safe technology and losses ηi_L otherwise, i.e. the audit benefit/cost scales with investment. All the entries in the safe firm branch follow this basic rule. For the risky firm, payoffs depend on the type of report given. For instance, after the node labelled (1) a successful risky firm that declares its true income would lose τBi to the tax authority if it escapes auditing and an extra ηi_L if it is audited. After node (2), a false report followed by an audit loses the firm $(\tau + s)Bi$ in addition to its audit response cost ηi_L while the tax authority gains the same amount less its auditing cost ζ .

Subgame: Risky Firm Report Figure A2 shows the subgame between the risky firm and auditor. Nature moves first, choosing if the firm fails (FAIL) or succeeds (SUC.). Once the outcome is realized, the firm chooses whether to report truthfully (TRUTH) or falsely (FALSE). The auditor then moves, choosing to audit (A.) or not to audit (N.). There is an information set around the auditor’s move as she doesn’t know if the firm is risky or safe. Since we assume that the auditor can observe the new capital produced by a successful firm, her optimal strategy is to play not audit (N.) if the firm fails. If the firm is successful, the auditor doesn’t have any dominant strategy. The nodes labelled (1) and (2) represents the auditor-firm subgame when the firm is successful. Since the auditor does not know if the successful firm is safe or risky, she faces the normal form game given in Figure A3 below. Since the auditor always plays a mixed strategy, the successful risky firm will play false if the following condition holds:

$$R^b + Bi - y([\tau + s]Bi + \eta i_L) > R^b + (1 - \tau)Bi - y\eta i_L \quad \equiv \quad \boxed{y < \frac{\tau}{\tau + s}}.$$

For the United States and many industrial economies, this condition is very likely to hold. The condition implies that a successful risky firm always reports falsely.

¹⁴In Graham et al. (2008), when a firm is forced to issue an income restatement due to fraud, it experiences a 68% increase in its loan spread: the amount the borrower pays in basis points over LIBOR (London Interbank Offered Rate) or LIBOR equivalent. Across firms, income restatements increase the spread by 85 basis points (Table 2, page 49). In Guedhami and Pittman (2008) increasing the probability of an IRS audit from 19% to 33% reduces the spread by 25 basis points.

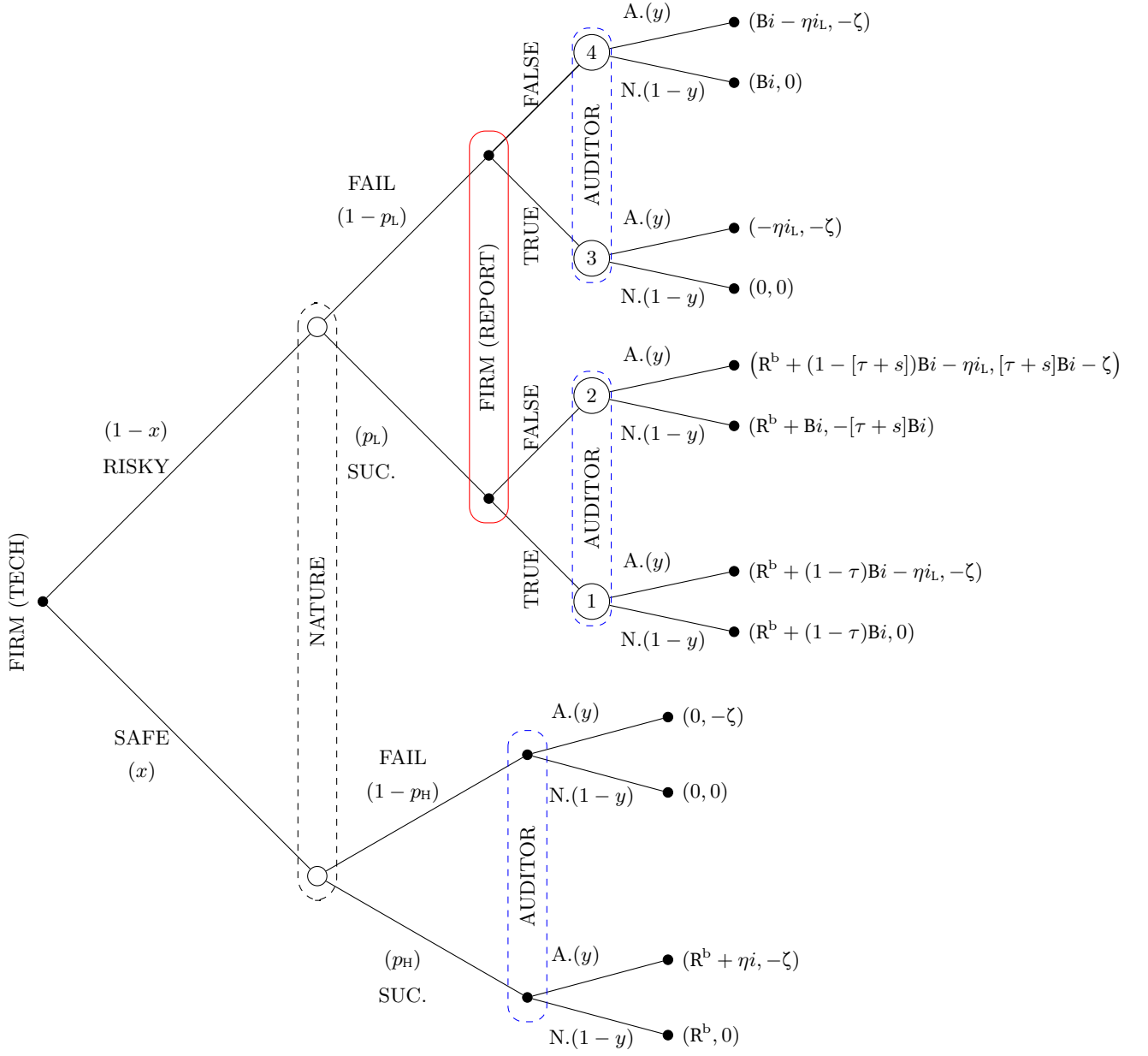


Figure A1: Extensive Form Game

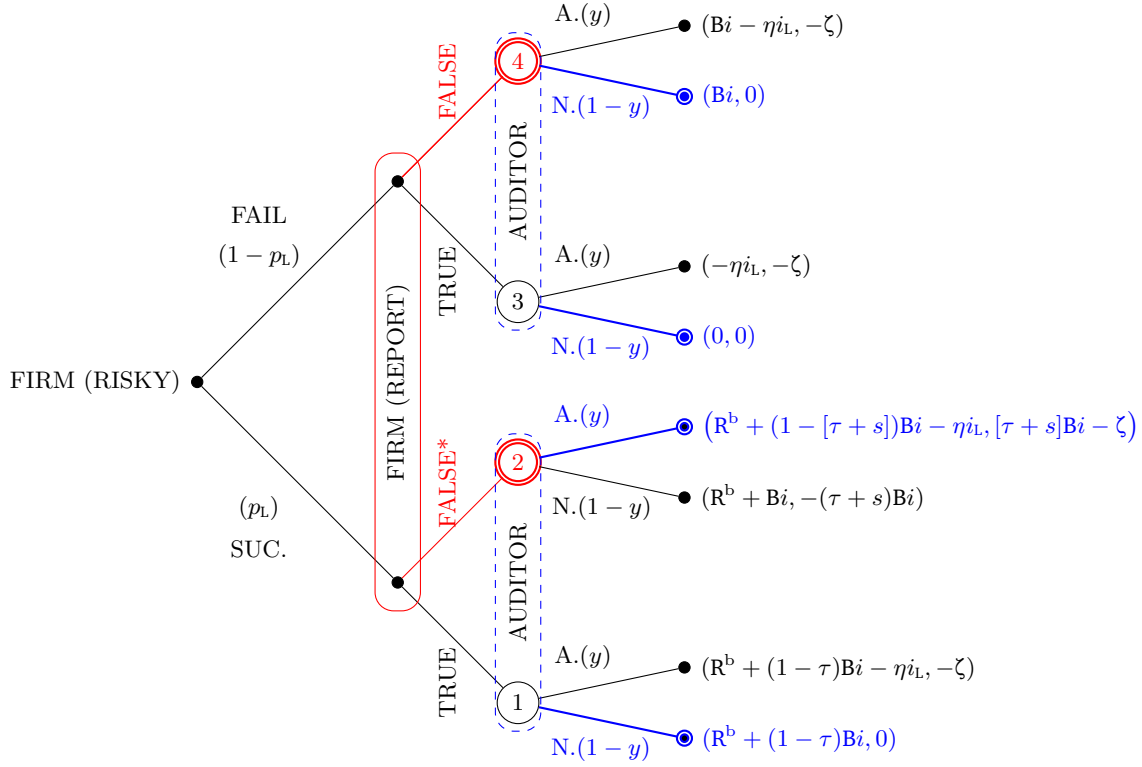
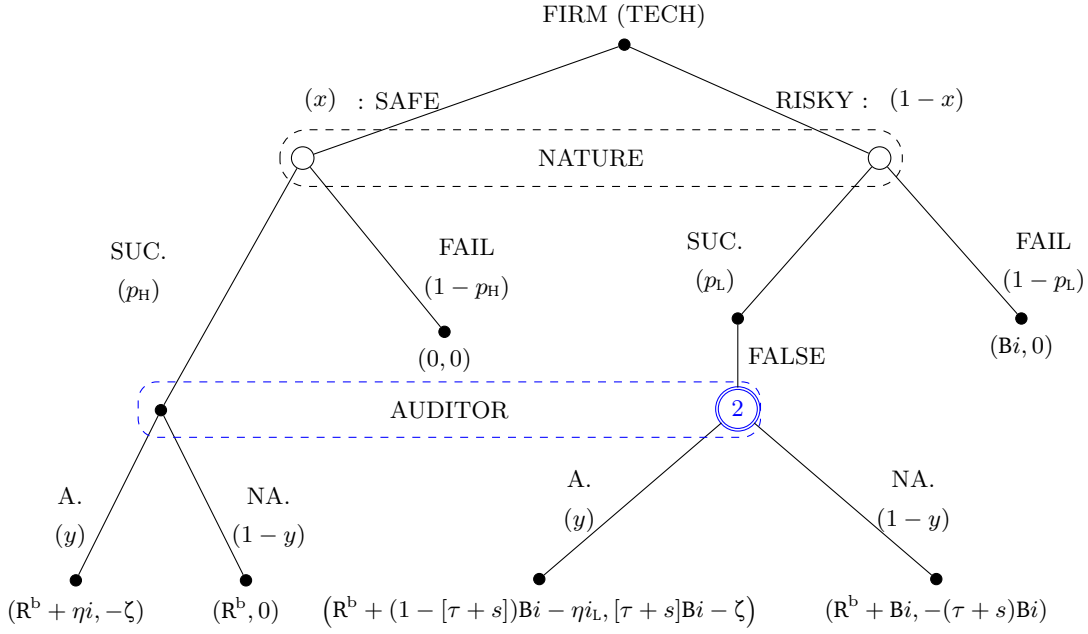


Figure A2: Subgame: Risky Firm

		Tax Authority	
		AUDIT (A)	NO AUDIT (N)
Firm	TRUE (T)	$R^b + (1 - \tau)Bi - \eta i_L$ $-\zeta$	$R^b + (1 - \tau)Bi$ 0
	FALSE (F)	$R^b + (1 - (\tau + s))Bi - \eta i_L$ $(\tau + s)Bi - \zeta$	$R^b + Bi$ $-(\tau + s)Bi$

Figure A3: Normal Form Representation of Successful Risky Firm vs. Auditor Game

Subgame: Technology Choice From the subgame in Figure A2, we know that the risky firm always plays FALSE and the tax authority/auditor never audits a failed firm. Therefore, the extensive form game in Figure A1 can be reduced the final game in Figure A4 which involves a SAFE/TRUE and RISKY/FALSE technology and report choice combination *for successful firms*, being played against an auditor who uses a mixed strategy of auditing (with probability y) or not (with probability $1 - y$).



		Tax Authority	
		AUDIT (y)	NO AUDIT (1 - y)
Firm	SAFE (x)	$p_H(R^b + \eta i)$ $- \zeta$	$p_H R^b$ 0
	RISKY (1 - x)	$p_L [R^b + (1 - (\tau + s))Bi - \eta i_L] + (1 - p_L)Bi$ $(\tau + s)Bi - \zeta$	$p_L R^b + Bi$ $-(\tau + s)Bi$

Figure A5: Normal Form Representation of Tech Choice with Auditing Game

This extensive form game can be represented in normal form as shown in Figure A5 below. From this representation, the expected payoff for an entrepreneur choosing the safe technology with probability x is:

$$E(x, y) = x \left\{ p_H \left(R^b + y\eta i \right) \right\} = (1 - x) \left\{ p_L \left[R^b + Bi - y \left((\tau + s)B + \eta(1 - B) \right) i \right] + (1 - p_L)Bi \right\}$$

Setting $\frac{dE(x, y)}{dy} = 0$ implies:

$$x = \frac{p_L \left[(\tau + s)B + \eta(1 - B) \right]}{p_L \left[(\tau + s)B + \eta(1 - B) \right] + p_H \eta} < 1$$

A.2 First Order Conditions

The first order conditions used in Sub-Section 2.3 are similar to those in the Kiyotaki and Moore (2012) model, but also with differences arising from the absence of labour income and money in our set-up. It is therefore necessary to flesh out how we obtain our optimality conditions. Recall that, *excluding storage*, the budget constraint, for an investing(borrower) and non-investing(lender) entrepreneur, are respectively:

$$c_t^b + i_t = [(1 - \tau)r_t + \tau\delta q_t + (1 - \delta)\phi_t q_t] a_t^b + \theta_t i_t \quad (\text{A.1})$$

$$c_t^l + q_t a_{t+1}^l = [(1 - \tau)r_t + \tau\delta q_t + q_t(1 - \delta)] a_t^l \quad (\text{A.2})$$

Solving (A.1) for investment i_t and substituting into the asset holding equation $a_{t+1}^b = (1 - \theta_t)[x p_H + (1 - x)p_L] R i_t + (1 - \phi_t)(1 - \delta)a_t^b$ gives the investing entrepreneur's consolidated budget constraint:

$$c_t^b + q_t^R a_{t+1}^b = \left[(1 - \tau)r_t + \tau\delta q_t + (1 - \delta) \left(\phi_t q_t + (1 - \phi_t) q_t^R \right) \right] a_t^b \quad (\text{A.3})$$

where $q_t^R = \frac{1}{[x p_H + (1 - x)p_L] R}$ is the effective replacement cost of equity for an investing entrepreneur as described before.

Letting the superscript $i, j = b, l$ tag variables for an agent who is of type i in period $t - 1$ and type j in period t . For instance, date t consumption of an agent who was a borrower in the previous period and is currently a lender is denoted by: c_t^{bl} . The Lagrangians for each of the four agent types are given by:

$$\begin{aligned} \mathcal{L}^l = & u(c_t^l) - \lambda_t^l \left[c_t^l + q_t a_{t+1}^l - [(1 - \tau)r_t + \tau\delta q_t + (1 - \delta)q_t] a_t^l \right] + \pi \beta E_t \left\{ u(c_{t+1}^b) \right. \\ & \left. - \lambda_{t+1}^b \left[c_{t+1}^b + q_{t+1}^R a_{t+2}^b - [(1 - \tau)r_{t+1} + \tau\delta q_{t+1} + (1 - \delta) \left(\phi_{t+1} q_{t+1} + (1 - \phi_{t+1}) q_{t+1}^R \right)] a_{t+1}^b \right] \right\} \\ & + (1 - \pi) \beta E_t \left\{ u(c_{t+1}^l) - \lambda_{t+1}^l \left[c_{t+1}^l + q_{t+1} a_{t+2}^l - [(1 - \tau)r_{t+1} + \tau\delta q_{t+1} + (1 - \delta)q_{t+1}] a_{t+1}^l \right] \right\} + \dots \end{aligned} \quad (\text{A.4})$$

$$\begin{aligned}
\mathcal{L}^{lb} = & u(c_t^{lb}) - \lambda_t^{lb} \left[c_t^{lb} + q_t^R a_{t+1}^b - [(1-\tau)r_t + \tau\delta q_t + (1-\delta)(\phi_t q_t + (1-\phi_t)q_t^R)]a_t^l \right] + \pi\beta\mathbf{E}_t \left\{ u(c_{t+1}^{bb}) \right. \\
& - \lambda_{t+1}^{bb} \left[c_{t+1}^{bb} + q_{t+1}^R a_{t+2}^b - [(1-\tau)r_{t+1} + \tau\delta q_{t+1} + (1-\delta)(\phi_{t+1}q_{t+1} + (1-\phi_{t+1})q_{t+1}^R)]a_{t+1}^b \right] \left. \right\} \\
& + (1-\pi)\beta\mathbf{E}_t \left\{ u(c_{t+1}^{bl}) - \lambda_{t+1}^{bl} \left[c_{t+1}^{bl} + q_{t+1}^l a_{t+2}^l - [(1-\tau)r_{t+1} + \tau\delta q_{t+1} + (1-\delta)q_{t+1}]a_{t+1}^b \right] \right\} + \dots
\end{aligned} \tag{A.5}$$

$$\begin{aligned}
\mathcal{L}^{bb} = & u(c_t^{bb}) - \lambda_t^{bb} \left[c_t^{bb} + q_t^R a_{t+1}^b - [(1-\tau)r_t + \tau\delta q_t + (1-\delta)(\phi_t q_t + (1-\phi_t)q_t^R)]a_t^b \right] + \pi\beta\mathbf{E}_t \left\{ u(c_{t+1}^{bb}) \right. \\
& - \lambda_{t+1}^{bb} \left[c_{t+1}^{bb} + q_{t+1}^R a_{t+2}^b - [(1-\tau)r_{t+1} + \tau\delta q_{t+1} + (1-\delta)(\phi_{t+1}q_{t+1} + (1-\phi_{t+1})q_{t+1}^R)]a_{t+1}^b \right] \left. \right\} \\
& + (1-\pi)\beta\mathbf{E}_t \left\{ u(c_{t+1}^{bl}) - \lambda_{t+1}^{bl} \left[c_{t+1}^{bl} + q_{t+1}^l a_{t+2}^l - [(1-\tau)r_{t+1} + \tau\delta q_{t+1} + (1-\delta)q_{t+1}]a_{t+1}^b \right] \right\} + \dots
\end{aligned} \tag{A.6}$$

$$\begin{aligned}
\mathcal{L}^{bl} = & u(c_t^{bl}) - \lambda_t^{bl} \left[c_t^{bl} + q_t a_{t+1}^l - [(1-\tau)r_t + \tau\delta q_t + (1-\delta)q_t]a_t^l \right] + \pi\beta\mathbf{E}_t \left\{ u(c_{t+1}^{lb}) \right. \\
& - \lambda_{t+1}^{lb} \left[c_{t+1}^{lb} + q_{t+1}^R a_{t+2}^b - [(1-\tau)r_{t+1} + \tau\delta q_{t+1} + (1-\delta)(\phi_{t+1}q_{t+1} + (1-\phi_{t+1})q_{t+1}^R)]a_{t+1}^l \right] \left. \right\} \\
& + (1-\pi)\beta\mathbf{E}_t \left\{ u(c_{t+1}^{ll}) - \lambda_{t+1}^{ll} \left[c_{t+1}^{ll} + q_{t+1}^l a_{t+2}^l - [(1-\tau)r_{t+1} + \tau\delta q_{t+1} + (1-\delta)q_{t+1}]a_{t+1}^l \right] \right\} + \dots
\end{aligned} \tag{A.7}$$

The first order conditions with respect to c_t^{ij}, a_{t+1}^j for $i, j = \{l, b\}$ are given by:

$$\frac{\partial \mathcal{L}^{ll}}{\partial c_t^{ll}} = \frac{1}{c_t^{ll}} - \lambda_t^{ll} = 0 \quad (\text{A.8})$$

$$\begin{aligned} \frac{\partial \mathcal{L}^{ll}}{\partial a_{t+1}^l} = & -\lambda_t^{ll} q_t + \pi \beta \mathbf{E}_t \left\{ \lambda_{t+1}^{lb} \left[(1-\tau)r_{t+1} + \tau \delta q_{t+1} + (1-\delta) \left(\phi_{t+1} q_{t+1} + (1-\phi_{t+1}) q_{t+1}^R \right) \right] \right\} \\ & + (1-\pi) \beta \mathbf{E}_t \left\{ \lambda_{t+1}^{ll} \left[(1-\tau)r_{t+1} + \tau \delta q_{t+1} + (1-\delta) q_{t+1} \right] \right\} = 0 \end{aligned}$$

$$\frac{\partial \mathcal{L}^{ll}}{\partial c_t^{lb}} = \frac{1}{c_t^{lb}} - \lambda_t^{lb} = 0 \quad (\text{A.9})$$

$$\begin{aligned} \frac{\partial \mathcal{L}^{lb}}{\partial a_{t+1}^b} = & -\lambda_t^{lb} q_t^R + \pi \beta \mathbf{E}_t \left\{ \lambda_{t+1}^{bb} \left[(1-\tau)r_{t+1} + \tau \delta q_{t+1} + (1-\delta) \left(\phi_{t+1} q_{t+1} + (1-\phi_{t+1}) q_{t+1}^R \right) \right] \right\} \\ & + (1-\pi) \beta \mathbf{E}_t \left\{ \lambda_{t+1}^{bl} \left[(1-\tau)r_{t+1} + \tau \delta q_{t+1} + (1-\delta) q_{t+1} \right] \right\} = 0 \end{aligned}$$

$$\frac{\partial \mathcal{L}^{bb}}{\partial c_t^{bb}} = \frac{1}{c_t^{bb}} - \lambda_t^{bb} = 0 \quad (\text{A.10})$$

$$\begin{aligned} \frac{\partial \mathcal{L}^{bb}}{\partial a_{t+1}^b} = & -\lambda_t^{bb} q_t^R + \pi \beta \mathbf{E}_t \left\{ \lambda_{t+1}^{bb} \left[(1-\tau)r_{t+1} + \tau \delta q_{t+1} + (1-\delta) \left(\phi_{t+1} q_{t+1} + (1-\phi_{t+1}) q_{t+1}^R \right) \right] \right\} \\ & + (1-\pi) \beta \mathbf{E}_t \left\{ \lambda_{t+1}^{bl} \left[(1-\tau)r_{t+1} + \tau \delta q_{t+1} + (1-\delta) q_{t+1} \right] \right\} = 0 \end{aligned}$$

$$\frac{\partial \mathcal{L}^{bl}}{\partial c_t^{bl}} = \frac{1}{c_t^{bl}} - \lambda_t^{bl} = 0 \quad (\text{A.11})$$

$$\begin{aligned} \frac{\partial \mathcal{L}^{bl}}{\partial a_{t+1}^l} = & -\lambda_t^{bl} q_t + \pi \beta \mathbf{E}_t \left\{ \lambda_{t+1}^{lb} \left[(1-\tau)r_{t+1} + \tau \delta q_{t+1} + (1-\delta) \left(\phi_{t+1} q_{t+1} + (1-\phi_{t+1}) q_{t+1}^R \right) \right] \right\} \\ & + (1-\pi) \beta \mathbf{E}_t \left\{ \lambda_{t+1}^{ll} \left[(1-\tau)r_{t+1} + \tau \delta q_{t+1} + (1-\delta) q_{t+1} \right] \right\} = 0 \end{aligned} \quad (\text{A.12})$$

Replacing for the λ_t^{ij} s and λ_{t+1}^{ij} s with the corresponding $\frac{1}{c_t^{ij}}$ s and $\frac{1}{c_{t+1}^{ij}}$ s gives the optimality conditions (13).