Estimating financial frictions under adaptive learning*

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Abstract

The paper studies the quantitative implications of priors beliefs and confidence in the model with learning. We introduce a constant-gain adaptive learning into a medium scale DSGE model with credit-constrained agents. We estimate the model both under rational expectations and adaptive learning using Bayesian techniques and study to what extent prior beliefs determine the evolution of expectations and endogenous variables. We analyze how the introduction of new macroeconomic policies is affected by priors, their precision, and expectations related to these policies.

Keywords: Expectations, Learning, Prior beliefs, Collateral Constraints, Bayesian econometrics

JEL classification: E32, E44, D83, C11

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1 Introduction

The collapse of housing market in the second half of 2000s triggered Global Financial Crisis. The crisis underscored the importance of the role that financial markets (and housing market in particular) play in the economy and brought back the interest in understanding the links between these markets and the macroeconomy. Pintus and Suda (2019) shows how the interaction of financial markets and learning could partially explain the crisis and its magnitude. Given their results it is natural to consider a model featuring financial markets in the quantitative evaluation of the importance of prior beliefs.

The recent literature introduces credit-constrained agents and house or land as collateral and achieves to explain positive co-movements between house prices and consumption or land prices and investment observed in data (Iacoviello and Neri, 2010; Liu et al., 2013; Guerrieri and Lorenzoni, 2017). The prevalent assumption in most models examining the nexus of housing (or financial) market and the macroeconomy is rational expectations. It implies implicitly that economic agents know (or their actions are consistent with) the exact knowledge of the structural form, the parameters and the stochastic structure of the economy. However, endowing agents with such knowledge may be unrealistic since evidence from forecasting surveys (Coibion and Gorodnichenko, 2015; Pancrazi and Pietrunti, 2018) and laboratory experiments (Hommes, 2013, Pfajfar and Žakelj, 2018) often suggest deviation from rationality. While in some models agents are allowed not to be fully rational (e.g. Pintus and Suda, 2019) there are hardly any studies that perform quantitative evaluation of the importance of this assumption.

Im this paper, we replace rational expectation with a constant-gain adaptive learning in a model of Iacoviello and Neri, 2010. Since we want to study the effects of learning on the economy through the housing market we want a quantitative model of the economy that (i) features a housing market, and (ii) is rich enough so we can distinguish alternative channels of interactions between housing and broad economy.

The rational expectation version of that model explains both qualitatively and quantitatively both the trends in the economy as well as in real housing prices and investment. Moreover, since Iacoviello and Neri (2010) focus on collateral effects on household borrowing given the finding in Pintus and Suda (2019) this model is well suited for quantitative evaluation.

In that model the economy consists of heterogeneous in discount factors households who derive utility from consumption, housing and leisure on the demand side and from housing and nonhousing sectors on the supply side. Impatient households' borrowing capacity is limited by the value of collateral given by expected value of their houses.

In our paper we relax the assumption of rational expectations and assume instead that while the agents form expectations using correctly specified economic models, they do not have perfect knowledge about the model parameters but rather use historical data to learn these parameters. In other words, agents are uncertain about the "true" parameters governing the law of motion of the economy but they update their beliefs about these parameters when new data arrives. The recent work finds that such setting results in improvement of model-fit to data (Milani, 2007; Slobodyan and Wouters, 2012b) and the ability to capture survey forecasts of macroeconomic aggregates (Ormeño and Molnár, 2015).

Even though our model is similar to the existing models that use housing or land as collateral, the key difference in our model is that the agents behave as econometrician and form expectations of future macroeconomic variables as a linear functions of past model variables. As new data become available every period, the agents update the coefficients of these linear functions using constant-gain recursive learning algorithm. Hence, the expectations of the agents depend on these time-varying coefficients that represents beliefs, and we insert these expectations into the structural model. This method has two important implications; first, depending on the size of the gain agents may have long memory in endogenous variables that creates persistence in long-run and second, the interconnectedness of structural parameters through the non-linear cross-equation restrictions may be significantly altered.

We estimate the model both under rational expectations and adaptive learning using Bayesian techniques. We employ Random Walk Metropolis Hasting algorithm. While there are papers that estimate a DSGE model under learning our is the first paper that estimates a model with collateral constraint.

First, we assess the joint role of financial frictions, collateral constraints and the departure from the full rationality assumption in explaining both the regular pattern of the US business cycle as well as recent financial crisis. We evaluate and compare the model fit and estimated parameters and the transmission mechanism in models with Rational Expectations and adaptive learning. Then, we consider how the assumptions about priors and their variance describing the adaptive learning matters for both parameter estimates as well as their revision.

Our results indicate that the dynamics of the economy under adaptive is different from the dynamics under rational expectations. We show that it is the prior beliefs and associated with them confidence that can determines the initial response of the economy. We confirm the results from Pintus, Suda and Turgut in the medium-sized quantitative models and find the similar pattern in which the more diffused priors are and the less agents "trusts" these priors the bigger revisions of agents' beliefs and the larger impact on endogenous variables.

1.1 Literature

Our paper relates to several strands of the literature.

This paper is closely related to the literature that estimates models with adaptive learning. Milani (2007) was the first to estimate a small New Keyesian model with adaptive learning. Slobodyan and Wouters (2012a, 2012b) are another examples of the estimation of medium scale model that relaxes the assumption of rational expectations and uses constant gain learning as belief-forming mechanism. They show that learning can act as an amplification mechanism and that parameters as well as the stochastic properties of underlying shocks are different under learning than under full-information rational expectations. More recent work includes Aguilar and Vázquez (2019) that builds on Slobodyan and Wouters (2012a) and introduce the term structure of interest rates. In our paper, we estimate the model with collateral constraint.

Our main result that agents overestimate the persistence of the leverage shock has been found in survey's expectations. Bordalo et al. (2020) document over-reaction in professional forecasters' expectations of macroeconomic outcomes.

In our model the key relationship is between changes in the leverage / house prices and agents decisions. Bailey et al. (2019) study the relationship between homebuyers' beliefs about future house price changes and their mortgage leverage choices. They work focus, however, on the role of heterogeneous beliefs in explaining households' financial decisions.

We look at the housing market as housing prices are among the most salient prices in the economy and, as shown by Chahrour and Gaballo (2019), house prices experiences are strongly correlated with the expectations regarding future income.

Our main assumption is that agents' expectations may no be the same as full information rational expectations (FIRE). There is a vast literature that questions such assumption.[1] Hey (1994) rejects rational expectations and finds evidence that adaptive expectations have explanatory power for belief dynamics. Contrary to these papers we focus on the importance of priors for the subsequent learning process. Not only we look

[1]See the survey in Manski (2018) on measuring expectations and confronting the empirical evidence with theory of rational expectations.

at the evolution of beliefs related to the law of motion of the economy but also study how much these beliefs change. here more standard version of adaptive learning that assumes less sophistication on the part of the agents.

Coibion et al. (2018) survey empirical micro-evidence that are at odds with the full-information rational expectation assumption. Moreover, using the Phillips curve they show how incorporating survey data on inflation expectations can address a number of otherwise puzzling shortcomings that arise under the assumption of full-information rational expectations.

2 Model

This section describes the model. We follow the Iacoviello and Neri (2010) since their framework allows for analysis of a variety of shocks: from housing to productivity to nominal rigidities but extend it with respect to expectation formation process by allowing for non-rational expectations and learning. In particular, we relax the assumption of rational expectations and endow agents with adaptive learning—constat gain version of recursive least squares learning—to form their expectations.

Below we present the sketch of the model resorting the reader to the appendix for the more detailed description.

3 Set-up

The model features two sectors — housing and non-housing — and two types of households: patient (lenders) and impatient (borrowers). Whereas all households work, consume and accumulate housing, it is patient households that own capital which they rent to firms and lend to impatient households.

3.0.1 Households

Impatient household chooses the level od consumption, c_t, (subject to internal habit formation), the amount of housing, h_t and supply of labor (to both housing and non-housing sectors: $n_{h,t}$ and $n_{c,t}$, respectively) to maximize a expected lifetime utility

$$E_0^* \sum_{t=0}^{\infty} \left(\beta^b G_c\right)^t z_t \left[\Gamma_c^b \ln(c_t^b - \epsilon^b c_{t-1}^b) + j_t \ln h_t^b - \frac{\tau_t}{1+\eta} \left((n_{c,t}^b)^{1+\xi} + (n_{h,t}^b)^{1+\xi}\right)^{\frac{1+\eta}{1+\xi}}\right] \quad (1)$$

where superscript b denotes variables and parameters that are specific to impatient household. β^b is a discount factor, z_t, τ_t, j_t are respectively shocks to intertemporal preferences, labor supply, and housing preferences, ϵ^l measures habits in consumption, G_c is the growth rate of consumption along the balanced growth path, ξ, η capture substitutability across sectors and disutility of labor , and Γ_c^b is a scaling factor.[2]

The budget constraint is given by

$$c_t^b + q_t h_t^b - b_t^b = \frac{w_{c,t} n_{c,t}^b}{X_{wc,t}} + \frac{w_{h,t} n_{h,t}^b}{X_{wh,t}} + q_t(1-\delta_h) h_{t-1}^b - \frac{R_{t-1} b_{t-1}^b}{\pi_t} + Div_t^b. \tag{2}$$

The borrowing constraint takes form

$$b_t^b \leq m_t E_t \left(\frac{q_{t+1} h_t^b \pi_{t+1}}{R_t} \right), \tag{3}$$

where m_t is stochastic (so time-varing) loan-to-value ratio (or leverage) and $E_t^* \left(\frac{q_{t+1} h_t^b \pi_{t+1}}{R_t} \right)$ is the present discounted value of their home.

Similarly, patient households chooses the level of consumption subject to internal habit formation, supply of labor to both sectors, and the amount of housing to maximize a expected lifetime utility

$$E_0^* \sum_{t=0}^{\infty} \left(\beta^l G_c\right)^t z_t \left[\Gamma_c^l \ln(c_t^l - \epsilon^l c_{t-1}^l) + j_t \ln h_t^l - \frac{\tau_t}{1+\eta} \left((n_{c,t}^l)^{1+\xi} + (n_{h,t}^l)^{1+\xi} \right)^{\frac{1+\eta}{1+\xi}}) \right]$$

where superscript l denotes variables and parameters that are specific to patient households (in equilibrium they are lenders) with the discount factor $\beta^l > \beta^b$.

Patient households maximize their utility subject to the budget constraint

$$\begin{aligned} c_t^l + k_{h,t} + k_{b,t} + \frac{k_{c,t}}{A_{k,t}} + q_t h_t + p_{l,t} l_t - b_t = \frac{w_{c,t} n_{c,t}^l}{X_{wc,t}} + \frac{w_{h,t} n_{h,t}^l}{X_{wh,t}} \\ + \left(R_{c,t} z_{c,t} + \frac{1-\delta_{kc}}{A_{k,t}} \right) k_{c,t-1} + \left(R_{h,t} z_{h,t} + 1 - \delta_{kc}\right) k_{h,t-1} + p_{b,t} k_{b,t} - \frac{R_{t-1} b_{t-1}}{\pi_t} \\ + \left(p_{l,t} + R_{l,t}\right) l_{t-1} + q_t (1-\delta_h) h_{t-1} + Div_t - \phi_t - \frac{a(z_{c,t}) k_{c,t-1}}{A_{k,t}} - a(z_{h,t}) k_{h,t-1} \end{aligned} \tag{4}$$

[2]As pointed out by Iacoviello and Neri (2010) for $\Gamma_c^i = \frac{G_c - \epsilon^i}{G_c - \beta^i \epsilon^i G_c}$ the marginal utility of consumption equals $\frac{1}{c^i}$ in the steady state for $i = b, l$.

3.0.2 Production

There are two sectors in the economy: housing and non-housing sectors. The former operates in competitive market with flexible prices. The production process in the housing sector utilizes labor and capital (just as in case of nonhousing goods) as well as land, l_t, and intermediate inputs $k_{b,t}$ according to the following technology

$$IH_t = \left(A_{h,t}(n^l_{h,t})^{\alpha}(n^b_{h,t})^{1-\alpha}\right)^{1-\mu_h-\mu_b-\mu_l}(z_{h,t}k_{h,t-1})^{\mu_h}k_{b,t}^{\mu_b}l_t^{\mu_l} \tag{5}$$

where $A_{h,t}$ measures productivity in non-housing sector. Note that by modeling housing sector as wholesale market, housing features flexible prices.[3]

The non-housing sector covers the production of consumption capital good, capital goods, as well as intermediate input. The production process has two steps with first the wholesale goods being produced in competitive (flexible price) market, and then final goods being produced as the composite retail goods (each being differentiated wholesale good).

The wholesale goods, Y_t, are produced according to Cobb-Douglas production function with labor, $l^i_{c,t}$, and capital, $k_{c,t}$ as inputs according to

$$Y_t = \left(A_{c,t}(n^l_{c,t})^{\alpha}(n^b_{c,t})^{1-\alpha}\right)^{1-\mu_c}(z_{c,t}k_{c,t-1})^{\mu_c} \tag{6}$$

where $n^l_{c,t}$ and $n^l_{c,t}$ are labor inputs into the nonhousing sector (c) for patient (l) and impatient (b) households, $A_{c,t}$ is labour-augmenting productivity, $z_{c,t}$ is capital specific productivity shock, and $k_{c,t-1}$ denotes the capital in that sector.

Final consumption good is produced in monopolistically competitive retail consumption sector. Retailers purchase wholesale goods Y_t which at price P^w_t, differentiate them, and then sell them at a markup $X_t = P_t/P^w_t$ to the households who CES aggregate them into final consumption/investment/intermediate good.

Wholesale firms purchase/hire inputs to produce consumption good and housing using the two technologies above to maximize profits

$$\max\left\{\frac{Y_t}{X_t} + q_t IH_t - \left(\sum_{i=b,l}\sum_{j=c,h} w_{j,t}h^i_{j,t} + \sum_{j=c,h} R_{i,t}z_{i,t}k_{i,t-1} + R_{l,t}l_{t-1} + p_{b,t}k_{b,t}\right)\right\} \tag{7}$$

[3] Barsky, House, and Kimball (2007) provide evidence supporting such assumption.

3.0.3 Price and wage stickiness

In the goods market we assume price rigidity a la Calvo with fraction θ_π of retailers not being able to optimally set prices each period but indexing the prices with previous period inflation rate with elasticity of ι_π.

The implied consumption-sector Phillips curve

$$\ln \pi_t - \iota_\pi \pi_{t-1} = \beta G_C (E_t^* \ln \pi_{t+1} - \iota_\pi \ln \pi_t) - \epsilon_\pi \ln(X_t/X) + u_{p,t} \tag{8}$$

Wages are also assumed to be sticky. Households "sell" there labor to unions (one union per every market-household type pair) who sell them at a premium to wholesale firms in consumption good and housing markets.

3.0.4 Policy

Monetary policy is set using Taylor-type rule with nominal interest rate responding with to inflation deviation from the target and to the rate of GDP growth,

$$R_t = R_{t-1}^{r_R} \pi_t^{(1-r_R)r_\pi} \left(\frac{GDP_t}{G_C GDP_{t-1}} \right)^{(1-r_R)r_Y} \overline{rr}^{(1-r_R)r_R} \frac{u_{R,t}}{s_t} \tag{9}$$

where $\overline{rr}$ denotes the steady-state real interest rate; s_t is stochastic deviations of inflation from the steady state, and u_t is monetary policy i.i.d. shock.

3.0.5 Market clearing conditions

In the market for houses the is given by

$$H_t = IH_t + (1 - \delta_h) H_{t-1} \tag{10}$$

where $H_t = h_t^l + h_t^b$ denotes stock of housing of borrowers and lenders in period t and IH_t denotes new homes.

The market clearing condition in the goods market is

$$Y_t = C_t + IK_{c,t}/A_{k,t} + IK_{h,t} + k_{b_t} + \phi_t \tag{11}$$

where $C_t = c_t^l + c_t^b$ is aggregate consumption.

Additionally, all four labor markets clear.

3.0.6 Equilibrium

In order to introduce adaptive learning modelled as constant gain or recursive least squares, we need to linearize the system around the balanced growth path. Appendix A contains log-linearization equations of the model with stochastic collateral.

Linearizing the first-order conditions and market clearing conditions around the steady state allows us to write down the system as a linearized expectational system, where all variables are expressed in percentage deviations from the steady state:

$$X_t = \mathbf{A}X_{t-1} + \mathbf{B}E_{t-1}[X_t] + \mathbf{C}E_t[X_{t+1}] + \mathbf{D}\xi_t, \tag{12}$$

where X_t is vector observed endogenous variables whereas ξ_t is not and $\mathbf{A}, \mathbf{B}, \mathbf{C}, \mathbf{D}$ are matrices expressed as functions of parameters.

3.1 Expectations

The key departure in our model from the Iacoviello and Neri (2010) is the formation of expectations. In this paper, similarly to Pintus and Suda (2019) we assume that instead of holding rational expectations all agents are forming expectations in adaptive way in the spirit of Marcet and Sargent (1989) and Evans and Honkapohja (2001). The linearized system is as in equation (12)

$$X_t = \mathbf{A}X_{t-1} + \mathbf{B}E^*_{t-1}[X_t] + \mathbf{C}E^*_t[X_{t+1}] + \mathbf{D}\xi_t \tag{13}$$

where the operator E^*_t indicates expectations that are taken using all information available at t but that are not rational. More precisely, agents behave as econometricians who adopt the following perceived law of motion (PLM thereafter):

$$X_t = \mathbf{M}X_{t-1} + \mathbf{G}\xi_t, \tag{14}$$

which agents use for forecasting. In particular, (14) yields $E^*_t[X_{t+1}] = \mathbf{M}_{t-1}X_t$ and $E^*_{t-1}[X_t] = \mathbf{M}_{t-2}X_{t-1}$. The actual law of motion (ALM thereafter) results from combining (13) and (14) which gives

$$[\mathbb{I} - \mathbf{C}\mathbf{M}_{t-1}]X_t = [\mathbf{A} + \mathbf{B}\mathbf{M}_{t-2}]X_{t-1} + \mathbf{D}\xi_t \tag{15}$$

When $\mathbf{M}$ coincides with $\mathbf{M}^{\mathbf{re}}$ derived above, then agents hold rational expectations. However, beliefs captured in $\mathbf{M}$ may differ temporarily from RE. Following Evans and

Honkapohja (2001) we assume that they are updated in real time using recursive learning algorithms. This means that the belief matrix $\mathbf{M}$ is time-varying and its coefficients are updated according to:

$$\mathbf{M}_t = \mathbf{M}_{t-1} + \nu \mathbf{R}_t^{-1} X_{t-1}(X_t - \mathbf{M}'_{t-1} X_{t-1}) \tag{16}$$

$$\mathbf{R}_t = \mathbf{R}_{t-1} + \nu (X_{t-1} X'_{t-1} - \mathbf{R}_{t-1)}, \tag{17}$$

where R is the estimate of the variance-covariance matrix and ν is a constant gain parameter.[4]

These two equations describe the law of motion of the beliefs. In this paper we will study how the dynamics of M are affected by the $\mathbf{M}_0$ and $\mathbf{R}_0$, which denote the initial (i.e. prior to seeing any data) perception of the economy.

The mapping from the PLM (in equation 14) into the ALM (equation 15) is given by:

$$T(\mathbf{M}) = [\mathbb{I} - \mathbf{CM}]^{-1} [\mathbf{A} + \mathbf{BM}] \tag{18}$$

The e-statbility of the system depends on the properties of the mapping $T(\mathbf{M})$ with the system being expectationally stable when all eigenvalues of $DT_M(\mathbf{M})$ have real parts less than 1 when evaluated at the fixed-point solutions of the T-map (18) that corresponds to RE.

4 Estimation

In this section we describe our estimation procedure and the data we use.

The technology and preference parameters of the model as well as parameters that govern the learning process are estimated using Bayesian method. For transformation of the variables we follow Iacoviello and Neri (2010). We cast the model in the state-space form which allows us, given the data and parameters, to compute the likelihood which, combined with prior distributions is used to estimate posteriors distribution. We use Metropolis-Hastings algorithm to do the latter.

The key difference between our paper and Iacoviello and Neri (2010) is the introduction of adaptive learning in place of rational expectations. Since the dynamics of the model are govern by equation (15) with the behavior of matrix M_t determined by equation (16) we have to address the potential non-linearity of the system. Recall that

[4]In general we could allow the gain to vary, ν_t, which for $\nu_t = 1/(t+1)$ would imply a least squares learning. In this paper we assume only a constant gain, however.

actual law of motion that describes the behavior of the endogenous variables is given by

$$[\mathbb{I} - \mathbf{C}\mathbf{M}_{t-1}]X_t = [\mathbf{A} + \mathbf{B}\mathbf{M}_{t-2}]X_{t-1} + \mathbf{D}\xi_t. \tag{19}$$

Since M_t is the function of $\{X_\tau\}_{\tau=0}^{t}$ it seems that we have non-linear system in X_t, which would make the use of Kalman Filter incorrect. However, given our informational assumptions it is not the case. We explicitly assume that agents can use only data that are available to them at the time of the forecast, i.e. at time t when computing the expected value $E_t X_{t+1}$ agents have only access to $\{X_1, \ldots, X_{t-1}\}$. Therefore, when computing $E_t X_{t+1}$ they use M_{t-1} that contains information until X_{t-1}.

Note also that since the behavior of matrix M_t is given by (16)

$$\mathbf{M}_t = \mathbf{M}_{t-1} + \nu \mathbf{R}_t^{-1} X_{t-1}(X_t - \mathbf{M}'_{t-1}X_{t-1}) \tag{16}$$

the behavior for M from the perspective of the Kalman Filter at t is fully determined by the data and past values of $\mathbf{M}_{t-1}$. The only new parameter introduced by adaptive learning is the gain, ν, and prior beliefs captured by $\mathbf{M}_0$ and $\mathbf{R}_0$. In this paper we calibrate the initial beliefs matrix $\mathbf{M}_0$ to correspond to rational expectation solution of the model and vary $\mathbf{R}_0$ to study the effects of confidence on the system.

Given the zero interest rate observed in the US from 2008q4, the data sample is set to 1975q1-2008q3. Following Iacoviello and Neri (2010) we use data on real consumption, real residential investment, real business investment, real house prices, nominal interest rate, inflation, hours and wage inflation in the consumption sector, hours and wage inflation in the housing sector. Since we also introduced a stochastic process for collateral we use the quarterly data on leverage from Boz and Mendoza (2014) and Pintus and Suda (2019). Since we explicitly account for trends in the model we first transform the data to obtain a trend stationary time series.[5]

5 Results

In this section we present the estimation results of the model with adaptive learning.

The model in this paper differs from Iacoviello and Neri (2010) along two dimensions. First, we allow the leverage to be stochastic process described by AR(1). Second, we replace the full information rational expectation agents with econometricians who re-

[5]Despite leverage increasing in the period we analyze, we consider it stationary. Pintus and Suda (2019) estimate the AR(1) process for leverage and confirm that it is stationary.

estimate their perceived law of motion every period the data becomes available. From the empirical perspective both of these elements changes the dynamics of the model and the estimates of the parameters. We we analyze these two elements separately.

5.1 Learning vs rational expectation without leverage shocks

First, we present the results comparing the effect of learning on parameter estimates in the model without stochastic leverage. In this section we treat $m_t = m$ as parameter that is estimated along all other parameters. Since we no longer treat leverage as a exogenous variable we do not use data on leverage. Table 1 presents the results.

Table 1. Posterior means of parameters for model with adaptive learning (AL) and rational expectations (RE) in the model without stochastic leverage

		Adaptive learning			RE		
Description	Parameter	Mean	5%	95%	Mean	5%	95%
Std. dev. of productivity shock in consumption sector	σ_{AC}	0.0119	0.0114	0.0126	0.0104	0.0094	0.0114
Std. dev. of monetary shock	σ_e	0.0026	0.0024	0.0029	0.0033	0.0028	0.0039
Std. dev. of productivity shock in housing sector	σ_{AH}	0.0194	0.0183	0.0210	0.0185	0.0169	0.0199
Std. dev. of productivity shock in non-residential sector	σ_{AK}	0.0065	0.0059	0.0072	0.0080	0.0063	0.0098
Std. dev. of housing preference shock	σ_j	0.0120	0.0111	0.0132	0.0119	0.0098	0.0153
Std. dev. of cost push-upshock	σ_p	0.0040	0.0036	0.0044	0.0042	0.0036	0.0049
Std. dev. of inflationary shock	σ_s	0.0232	0.0196	0.0259	0.0254	0.0191	0.0321
Std. dev. of labor shock	σ_τ	0.0387	0.0373	0.0402	0.0219	0.0158	0.0315
Std. dev. of intertemporal preference shock	σ_z	0.0115	0.0104	0.0125	0.0078	0.0057	0.0104
Noise in hours in housing	σ_{NH}	0.1109	0.1084	0.1126	0.1246	0.1128	0.1382
Noise in wage in housing	σ_{WH}	0.0080	0.0075	0.0085	0.0070	0.0063	0.0077
share of patient labor	α	0.9235	0.92	0.93	0.9273	0.91	0.95
habit formation for patient	ε	0.4957	0.48	0.51	0.3040	0.23	0.39
habit formation for impatient	ε'	0.6723	0.66	0.69	0.5542	0.45	0.67
disutility of labor patient	η	0.3863	0.36	0.41	0.4774	0.33	0.64
disutility of labor impatient	η'	0.9979	0.96	1.03	0.5346	0.37	0.73
capital adjustment costs	ψ_k	15.7253	15.05	16.38	14.3003	11.62	17.91
capital adjustment costs	ψ_h	12.1610	11.60	13.02	10.8114	7.21	15.37
inflation indexation	ι_p	0.4961	0.44	0.53	0.5430	0.42	0.70
wage indexation in consumption sector	$\iota_{w,c}$	0.0610	0.02	0.13	0.0853	0.03	0.16
wage indexation in housing sector	$\iota_{w,h}$	0.3964	0.35	0.45	0.3846	0.20	0.55
disutility of labor patient	ξ	-0.6921	-0.72	-0.66	-0.7451	-0.94	-0.56
disutility of labor impatient	ξ'	-0.9019	-0.92	-0.88	-0.9585	-1.14	-0.81
persistence productivity shock in consumption sector	ρ_{AC}	0.9882	0.98	0.99	0.9240	0.89	0.95
persistence productivity shock in housing sector	ρ_{AH}	0.9653	0.96	0.97	0.9786	0.97	0.99
persistence housing preference shock	ρ_j	0.9990	1.00	1.00	0.9749	0.96	0.99
persistence productivity shock in nonresidental sector	ρ_{AK}	0.8964	0.87	0.91	0.9174	0.89	0.94
persistence labor preference shock	ρ_r	0.8559	0.84	0.87	0.9200	0.89	0.95
persistence intertemporal preference shock	ρ_z	0.6580	0.63	0.68	0.6948	0.46	0.85
Taylor rule inflation feedback	R_p	1.5341	1.51	1.56	1.4001	1.29	1.54
Taylor rule AR parameter	R_r	0.7082	0.69	0.72	0.6122	0.54	0.67
Taylor rule output gap feedback	R_Y	0.4259	0.40	0.46	0.5332	0.44	0.63
Fraction of price non-optimizers	θ	0.8579	0.84	0.87	0.8661	0.83	0.89
Fraction of wage non-optimizers in consumption	$\theta_{w,c}$	0.8123	0.80	0.82	0.7650	0.72	0.81
Fraction of wage non-optimizers in housing	$\theta_{w,h}$	0.8966	0.89	0.91	0.9137	0.89	0.93
Trend in consumption	γ_{AC}	0.0033	0.00	0.00	0.0032	0.00	0.00
Trend in housing	γ_{AK}	0.0012	0.00	0.00	0.0006	0.00	0.00
Trend in nonresidential investment	γ_{AH}	0.0025	0.00	0.00	0.0028	0.00	0.00
capacity utilization curvature	ζ	0.8823	0.84	0.93	0.6339	0.50	0.78
Constant gain parameters	g	0.0164	0.0151	0.0178			
Log Likelihood			5032.4			5003.1	

Note:

Table 1

The direct comparison of columns 3 and 6 shows the effect of learning on parameter estimates in the model without collateral constraint.

5.2 Stochastic leverage under rational expectation

In this subsection we analyze the effects of stochastic leverage on the parameter estimates. Table 2 presents the results.

Parameter	with Leverage shocks	w/o Leverage shocks
σ_{AC}	0.0103	0.0098
σ_e	0.0026	0.0031
σ_{AH}	0.0203	0.0202
σ_{AK}	0.0908	0.0071
σ_j	0.0156	0.0122
σ_p	0.0042	0.0041
σ_s	0.0261	0.0297
σ_τ	0.0366	0.0225
σ_z	0.0155	0.0010
σ_m	0.0335	—
$\sigma_{n,h}$	0.1559	0.1872
$\sigma_{w,h}$	0.0056	0.0052
α	0.8301	0.9239
ϵ_c	0.3038	0.3106
ϵ_{c1}	0.6491	0.5756
η_c	0.4329	0.4837
η_{c1}	0.4786	0.4452
ψ_k	20.1533	16.6840
ψ_h	11.8288	10.1142
ι_π	0.6134	0.4903
$\iota_{w,c}$	0.1423	0.1733
$\iota_{w,h}$	0.2826	0.1407
ν_c	-1.0281	-0.9618
ν_{c1}	-0.9954	-0.9282
ρ_{AC}	0.9922	0.9905
ρ_{AH}	0.9971	0.9943
ρ_j	0.9729	0.9997
ρ_{AK}	0.9204	0.9110
ρ_τ	0.9217	0.9510
ρ_z	0.9995	0.9573
ρ_m	0.9550	—
R_π	1.4131	1.4196
R_R	0.6988	0.6653
R_Y	0.3374	0.5016
θ	0.8505	0.8799
$\theta_{w,c}$	0.9050	0.8610
$\theta_{w,h}$	0.9781	0.9713
γ_{AC}	0.0019	0.0017
γ_{AK}	0.0032	0.0004
γ_{AH}	0.0030	0.0032
ζ	0.9419	0.7463

Note: Stochastic leverage

Table 2: Posterior means of parameters for model with and w/o leverage shock. Both models are RE.

The introduction of stochastic leverage does affect estaimtes of some paramters. First, the persistence of leverage shocks is very high at 0.995 while the shocks are relatively small with $\sigma_m = 0.0335$.

5.3 Learning vs rational expectation leverage shock

Finally, we analyze the effects of learning in the model with stochastic collateral. In this section we allow m_t to follow and exogenous AR(1) process for which estimates mean and persistence. Since we now treat leverage as a exogenous variable we use data on leverage. Table 3 presents the results.

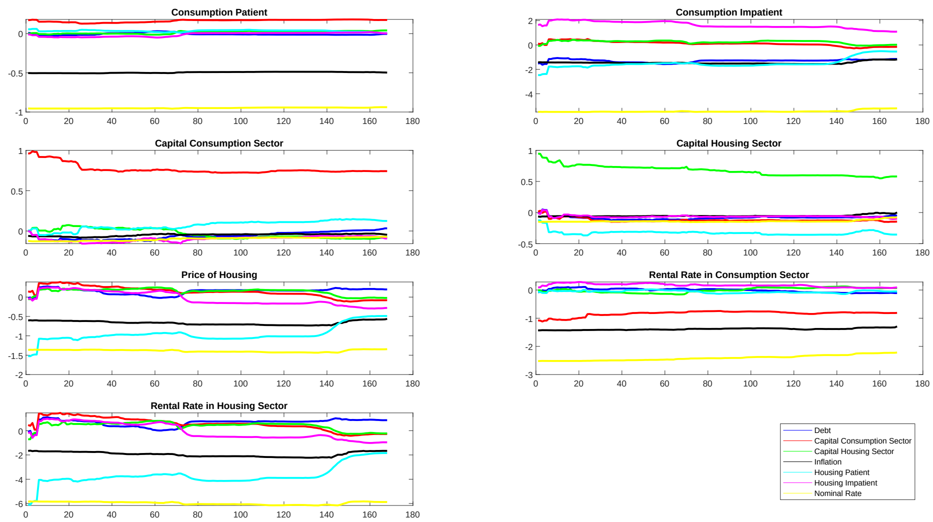
Parameter	Adaptive Learning	Rational Expectations
σ_z	0.0109	0.0137
σ_τ	0.0386	0.0496
σ_j	0.0352	0.1297
σ_m	0.0325	0.0335
σ_{AK}	0.0143	0.0160
σ_{AC}	0.0103	0.0103
σ_{AH}	0.0218	0.0220
σ_s	0.0155	0.0151
σ_e	0.0025	0.0025
σ_p	0.0047	0.0041
$\sigma_{n,h}$	0.1500	0.1656
$\sigma_{w,h}$	0.0057	0.0056
α	0.7889	0.8482
ϵ_c	0.4152	0.3378
ϵ_{c1}	0.6614	0.6729
η_c	0.2449	0.6241
η_{c1}	0.3671	0.4673
ψ_k	18.2925	20.9743
ψ_h	10.3278	9.8580
ι_π	0.6784	0.5963
$\iota_{w,c}$	0.1201	0.1358
$\iota_{w,h}$	0.2625	0.4320
ν_c	-0.9601	-1.1136
ν_{c1}	-0.8816	-0.8639
ρ_{AC}	0.9868	0.9932
ρ_{AH}	0.9976	0.9963
ρ_j	0.9988	0.9574
ρ_{AK}	0.9384	0.9321
ρ_τ	0.9142	0.9164
ρ_z	0.9372	0.9994
ρ_M	0.9338	0.9499
R_π	1.2688	1.2431
R_R	0.6764	0.7046
R_Y	0.3017	0.3345
θ	0.8343	0.8624
$\theta_{w,c}$	0.9059	0.9226
$\theta_{w,h}$	0.9798	0.9763
γ_{AC}	0.0019	0.0019
γ_{AH}	0.0023	0.0033
γ_{AK}	0.0033	0.0030
ζ	0.9288	0.9348
gain	0.0014	NaN

Note: Parameters for both models estimated with 200k draws of MH algorithm. 100k first draws burned.

Table 3: Posterior means of parameters under learning and RE for the model with leverage shock.

This comparison shows the combine effect of learning and the presence of collateral shocks on the estimates of the parameters.

Figure 1: Responses under learning in case of the underestimation of $\mathbf{M_0}$ under dispersed initial beliefs.



5.4 Stability of beliefs

In this section we repeat our exercise and compare the effects of confidence in the priors beliefs on revision of beliefs. Since we estimate the model the dynamics of observable variables are always the same. Since each time we simulate hte model we change the RE equilibrium we initialize matrix M at the corresponding (for each draw) M^{RE}. What is different, however, is the evolution of VAR(1) matrix M capturing the perceived law of motion.

We start with increasing the magnitude of the matrix R_0 to capture the effects of higher confidence in initial beliefs. Figure 1 shows the dynamics of coefficients in that case.

Compare this with the case of even more dispersed initial beliefs. Figure 2 presents the case of initial variance-covariance matrix R_0 being smaller than in the previous case.

If initial beliefs are strong, that is the diagonal elements of the variance covariance matrix R_0 are small , there is hardly any updating as shown in Figure 3. This implies that there is hardly any updating of prior beliefs even after observing the time series.

Figure 2: Responses under learning in case of the underestimation of $\mathbf{M_0}$ under very dispersed initial beliefs.

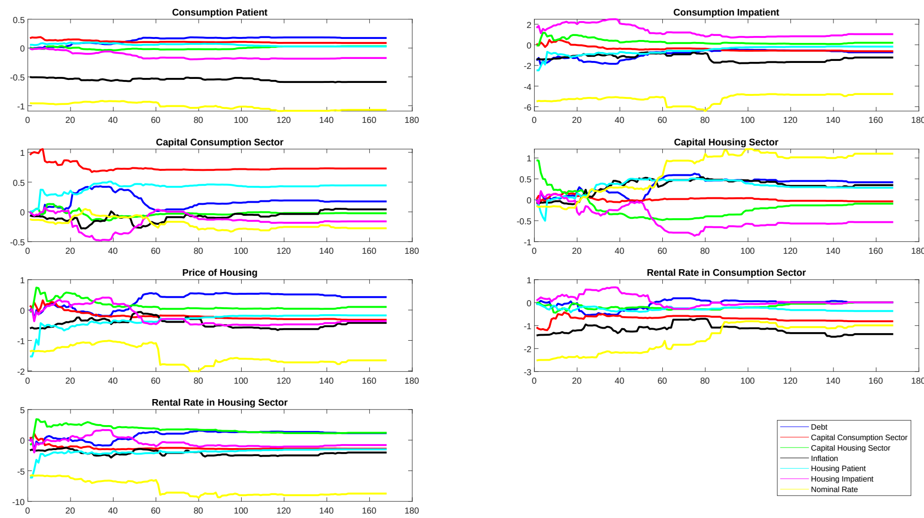
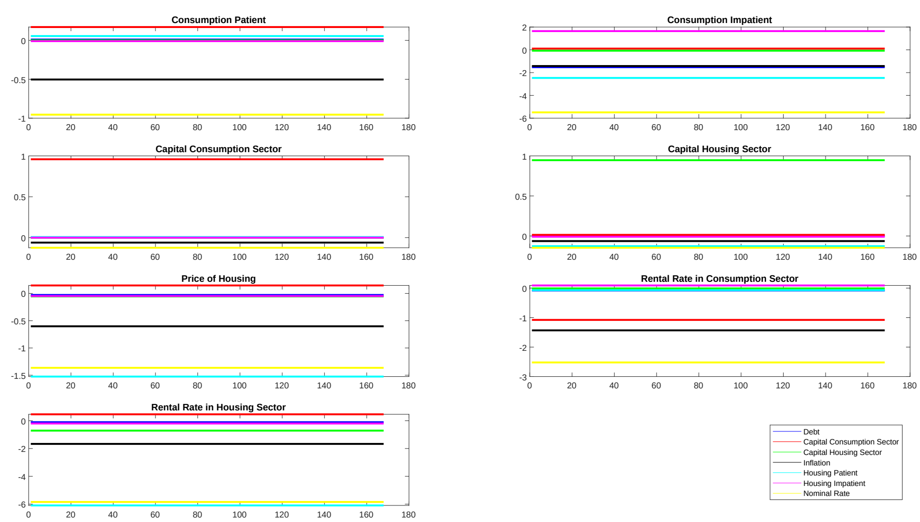


Figure 3: Responses under learning in case of the underestimation of $\mathbf{M_0}$ under strong initial beliefs.



6 Conclusions

In this paper we estimate a medium-sized DSGE model with the stochastic collateral constraint and learning. To that end we extend the Iacoviello and Neri (2010) with (i) the stochastic leverage that follows an AR(1) prices, and (ii) the constant gain adaptive learning. For the latter we relax the assumption of rational expectations by replacing the traditional full information rational expectation agents with econometricians.

We show how (i) stochastic leverage and (ii) adaptive learning affect the estimates of the structural parameters. We also show how the variance-covariance of priors affects the updating and dynamics of beliefs.

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Appendices

A Full model

A.1 Log-linearized equations

A.1.1 Main Variables

Budget constraint of the patient household:

$$
\begin{aligned}
\hat{c}_t + \frac{\tilde{k}_c}{\tilde{c}}\left(\hat{k}_{c,t} - \hat{a}_{k,t}\right) + \frac{\tilde{k}_h}{\tilde{c}}\hat{k}_{h,t} + \frac{\tilde{q}\tilde{h}}{\tilde{c}}\left(\hat{q}_t + \hat{h}_t\right) + \frac{\tilde{b}'}{\tilde{c}}\hat{b}'_t = {} & (1-\delta_h)\frac{\tilde{q}\tilde{h}}{\tilde{c}G_H}\left(\hat{q}_t + \hat{h}_{t-1}\right) \qquad (20)\\
& + \frac{\tilde{w}_c\tilde{n}_c}{\tilde{c}}\left(\hat{w}_{c,t} + \hat{n}_{c,t}\right) + \frac{\tilde{w}_h\tilde{n}_h}{\tilde{c}}\left(\hat{w}_{h,t} + \hat{n}_{h,t}\right)\\
& + \frac{\tilde{Y}}{\tilde{c}}\hat{Y}_t - \frac{\tilde{Y}}{\tilde{c}X}\left(\hat{Y}_t - \hat{X}_t\right) + \frac{\tilde{R}_c\tilde{k}_c}{\tilde{c}G_{KC}}\left(\hat{R}_{c,t} + \hat{z}_{c,t} + \hat{k}_{c,t-1}\right)\\
& + (1-\delta_{kc})\frac{\tilde{k}_c}{\tilde{c}G_{KC}}\left(\hat{k}_{c,t-1} - \hat{a}_{k,t}\right) + \frac{\tilde{R}_h\tilde{k}_h}{\tilde{c}G_{KH}}\left(\hat{R}_{h,t} + \hat{z}_{h,t} + \hat{k}_{h,t-1}\right)\\
& + (1-\delta_{kh})\frac{\tilde{k}_h}{\tilde{c}G_{KH}}\hat{k}_{h,t-1} + \frac{\tilde{R}\tilde{b}'}{\tilde{c}G_C}\left(\hat{R}_{t-1} + \hat{b}'_{t-1} - \hat{\pi}_t\right) + \frac{\tilde{R}_l}{\tilde{c}}\hat{R}_{l,t}
\end{aligned}
\qquad (21)
$$

FOC with respect to the h_t:

$$
\hat{q}_t + \hat{u}_{c,t} = \frac{\tilde{u}_h}{\tilde{q}\tilde{u}_c}\hat{u}_{h,t} + \beta G_Q(1-\delta_h)\left(\hat{q}_{t+1} + \hat{u}_{c,t+1}\right) \qquad (22)
$$

FOC with respect to b_t:

$$
\hat{u}_{c,t} = \hat{u}_{c,t+1} + \hat{R}_t - \hat{\pi}_{t+1} \qquad (23)
$$

FOC with respect to $k_{c,t}$:

$$
\begin{aligned}
\hat{u}_{c,t} - \hat{a}_{k,t} + \phi_{kc}\left(\hat{k}_{c,t} - \hat{k}_{c,t-1}\right) = {} & \frac{\beta\tilde{R}_c}{G_{AK}}\left(\hat{u}_{c,t+1} + \hat{R}_{c,t+1} + \hat{z}_{c,t+1}\right) + (1-\delta_{kc})\frac{\beta}{G_{AK}}\left(\hat{u}_{c,t+1} - \hat{a}_{k,t+1}\right)\\
& + \frac{\beta}{G_C}\phi_{kc}\left(\hat{k}_{c,t+1} - \hat{k}_{c,t}\right)
\end{aligned}
\qquad (24)
$$

FOC with respect to $k_{h,t}$:

$$\begin{aligned}\hat{u}_{c,t} + \phi_{kh}\left(\hat{k}_{h,t} - \hat{k}_{h,t-1}\right) =& \beta\tilde{R}_h\left(\hat{u}_{c,t+1} + \hat{R}_{h,t+1} + \hat{z}_{h,t+1}\right) + (1-\delta_{kh})\,\beta\hat{u}_{c,t+1} \\ &+ \beta G_{KH}\phi_{kh}\left(\hat{k}_{h,t+1} - \hat{k}_{h,t}\right)\end{aligned} \tag{25}$$

FOC with respect to $n_{c,t}$:

$$\hat{u}_{nc,t} = \hat{u}_{c,t} + \hat{w}_{c,t} - \hat{X}_{wc,t} \tag{26}$$

FOC with respect to $n_{h,t}$:

$$\hat{u}_{nh,t} = \hat{u}_{c,t} + \hat{w}_{h,t} - \hat{X}_{wh,t} \tag{27}$$

FOC with respect to $z_{c,t}$:

$$\hat{a}_{k,t} + \hat{R}_{c,t} = \frac{Z_{kc}}{1 - Z_{kc}}\hat{z}_{c,t} \tag{28}$$

FOC with respect to $z_{h,t}$:

$$\hat{R}_{h,t} = \frac{Z_{kc}}{1 - Z_{kc}}\hat{z}_{h,t} \tag{29}$$

FOC with respect to the l_t:

$$\hat{u}_{c,t} + \hat{p}_{l,t} = \beta G_C\left(\hat{u}_{c,t+1} + \hat{p}_{l,t+1}\right) + \beta\frac{\tilde{R}_l}{\tilde{p}_l}\left(\hat{u}_{c,t+1} + \hat{R}_{l,t+1}\right) \tag{30}$$

Budget constraint of the impatient household:

$$\begin{aligned}\hat{c}'_t + \frac{\tilde{q}\tilde{h}'}{\tilde{c}'}\left(\hat{q}_t + \hat{h}'_t\right) + \frac{\tilde{R}\tilde{b}'}{\tilde{c}'G_C}\left(\hat{R}_{t-1} + \hat{b}'_{t-1} - \hat{\pi}_t\right) =& (1-\delta_h)\frac{\tilde{q}\tilde{h}'}{\tilde{c}'G_H}\left(\hat{q}_t + \hat{h}'_{t-1}\right) + \frac{\tilde{w}'_c\tilde{n}'_c}{\tilde{c}'}\left(\hat{w}'_{c,t} + \hat{n}'_{c,t}\right) \\ &+ \frac{\tilde{w}'_h\tilde{n}'_h}{\tilde{c}'}\left(\hat{w}'_{h,t} + \hat{n}'_{h,t}\right) + \frac{\tilde{b}'}{\tilde{c}'}\hat{b}'_t\end{aligned} \tag{31}$$

Borrowing constraint:

$$\hat{b}'_t = \hat{m}_t + \hat{q}_{t+1} + \hat{h}'_t + \hat{\pi}_{t+1} - \hat{R}_t \tag{32}$$

FOC with respect to h'_t:

$$\hat{q}_t + \hat{u}_{c',t} = \frac{\tilde{u}_{h'}}{\tilde{q}\tilde{u}_{c'}}\hat{u}_{h',t} + \beta' G_Q(1-\delta_h)\left(\hat{q}_{t+1} + \hat{u}_{c',t+1}\right) + \frac{G_Q m\tilde{\lambda}}{R\tilde{u}_{c'}}\left(\hat{m}_t + \hat{\lambda}_t + \hat{q}_{t+1} + \hat{\pi}_{t+1} - \hat{R}_t\right) \tag{33}$$

FOC with respect to b'_t:

$$\hat{u}_{c',t} = \frac{\beta'}{\beta}\left(\hat{u}_{c,t+1} + \hat{R}_t - \hat{\pi}_{t+1}\right) + \left(1 - \frac{\beta'}{\beta}\right)\hat{\lambda}_t \tag{34}$$

FOC with respect to $n'_{c,t}$:

$$\hat{u}_{nc',t} = \hat{u}_{c',t} + \hat{w}'_{c,t} - \hat{X}'{}_{wc,t} \tag{35}$$

FOC with respect to $n'_{h,t}$:

$$\hat{u}_{nh',t} = \hat{u}_{c',t} + \hat{w}'_{h,t} - \hat{X}'_{wh,t} \tag{36}$$

Wholesale production technology:

$$\hat{Y}_t = (1 - \mu_c)\left(\hat{Z}_{c,t} + \alpha \hat{n}_{c,t} + (1 - \alpha)\,\hat{n}'_{c,t}\right) + \mu_c\left(\hat{z}_{c,t} + \hat{k}_{c,t-1}\right) \tag{37}$$

Housing technology:

$$\hat{H}_t = (1 - \mu_h - \mu_b - \mu_l)\left(\hat{Z}_{h,t} + \alpha \hat{n}_{h,t} + (1 - \alpha)\,\hat{n}'_{h,t}\right) + \mu_h\left(\hat{z}_{h,t} + \hat{k}_{h,t-1}\right) + \mu_b \hat{k}_{b,t} \tag{38}$$

Wholesale firm FOC with respect to $n_{c,t}$:

$$\hat{Y}_t - \hat{X}_t - \hat{n}_{c,t} = \hat{w}_{c,t} \tag{39}$$

Wholesale firm FOC with respect to $n'_{c,t}$:

$$\hat{Y}_t - \hat{X}_t - \hat{n}'_{c,t} = \hat{w}'_{c,t} \tag{40}$$

Housing firm FOC with respect to $n_{h,t}$:

$$\hat{q}_t + \hat{H}_t - \hat{n}_{h,t} = \hat{w}_{h,t} \tag{41}$$

Housing firm FOC with respect to $n'_{h,t}$:

$$\hat{q}_t + \hat{H}_t - \hat{n}'_{h,t} = \hat{w}'_{h,t} \tag{42}$$

Wholesale firm FOC with respect to $k_{c,t-1}$:

$$\hat{Y}_t - \hat{X}_t - \hat{k}_{c,t-1} = \hat{R}_{c,t} + \hat{z}_{c,t} \tag{43}$$

Housing firm FOC with respect to $k_{h,t-1}$:

$$\hat{q}_t + \hat{H}_t - \hat{k}_{h,t-1} = \hat{R}_{h,t} + \hat{z}_{h,t} \tag{44}$$

Housing firm FOC with respect to l_{t-1}:

$$\hat{q}_t + \hat{H}_t = \hat{R}_{l,t} \tag{45}$$

Housing firm FOC with respect to $k_{b,t-1}$:

$$\hat{q}_t + \hat{H}_t = \hat{k}_{b,t} \tag{46}$$

Philips curve:

$$\hat{\pi}_t - \iota_\pi \hat{\pi}_{t-1} = \beta \left(\hat{\pi}_{t+1} - \iota_\pi \hat{\pi}_t \right) - \varepsilon_\pi \hat{X}_t + u_{p,t} \tag{47}$$

Wage equations:

$$\left(1 + \beta G_C\right) \hat{w}_{c,t} + \left(1 + \beta G_C \iota_{wc}\right) \hat{\pi}_t - \hat{w}_{c,t-1} - \iota_{wc} \hat{\pi}_{t-1} = \beta G_C \left(\hat{w}_{c,t+1} - \hat{\pi}_{t+1} \right) - \varepsilon_{wc} \hat{X}_{wc,t} \tag{48}$$

$$\left(1 + \beta' G_C\right) \hat{w}'_{c,t} + \left(1 + \beta' G_C \iota_{wc}\right) \hat{\pi}_t - \hat{w}'_{c,t-1} - \iota_{wc} \hat{\pi}_{t-1} = \beta' G_C \left(\hat{w}'_{c,t+1} - \hat{\pi}_{t+1} \right) - \varepsilon'_{wc} \hat{X}'_{wc,t} \tag{49}$$

$$\left(1 + \beta G_C\right) \hat{w}_{h,t} + \left(1 + \beta G_C \iota_{wh}\right) \hat{\pi}_t - \hat{w}_{h,t-1} - \iota_{wh} \hat{\pi}_{t-1} = \beta G_C \left(\hat{w}_{h,t+1} - \hat{\pi}_{t+1} \right) - \varepsilon_{wh} \hat{X}_{wh,t} \tag{50}$$

$$\left(1 + \beta' G_C\right) \hat{w}'_{h,t} + \left(1 + \beta' G_C \iota_{wh}\right) \hat{\pi}_t - \hat{w}'_{h,t-1} - \iota_{wh} \hat{\pi}_{t-1} = \beta' G_C \left(\hat{w}'_{h,t+1} - \hat{\pi}_{t+1} \right) - \varepsilon'_{wh} \hat{X}'_{wh,t} \tag{51}$$

Taylor Rule:

$$\hat{R}_t = r_R \hat{R}_{t-1} + \left(1 - r_R\right) r_\pi \hat{\pi}_t + \left(1 - r_R\right) r_Y \left(G\hat{D}P_t - G\hat{D}P_{t-1} \right) + e_{R,t} - A_{s,t} \tag{52}$$

Market clearing for housing:

$$\frac{\tilde{h}}{\tilde{H}} \hat{h}_t + \frac{\tilde{h}'}{\tilde{H}} \hat{h}'_t - \left(1 - \delta_h\right) \left(\frac{\tilde{h}}{G_H \tilde{H}} \hat{h}_{t-1} + \frac{\tilde{h}'}{G_H \tilde{H}} \hat{h}'_{t-1} \right) = \hat{H}_t \tag{53}$$

A.1.2 Auxiliary Variables

Marginal utility of consumption of the patient household:

$$\left(G_C - \varepsilon\right) \left(1 - \beta \varepsilon\right) \hat{u}_{c,t} = \left(G_C - \varepsilon\right) \left(\hat{z}_t - \beta \varepsilon \hat{z}_{t+1} \right) - \left(G_C + \beta \varepsilon^2 \right) \hat{c}_t + \varepsilon \hat{c}_{t-1} + \beta \varepsilon G_C \hat{c}_{t+1} \tag{54}$$

Marginal utility of housing of the patient household:

$$\hat{u}_{h,t} = \hat{z}_t + \hat{j}_t - \hat{h}_t \tag{55}$$

Marginal disutility of working of the patient household in the consumption sector:

$$\hat{u}_{nc,t} = \hat{z}_t + \hat{\tau}_t + \xi \hat{n}_{c,t} + (\eta - \varepsilon)\left(\frac{n_c^{1+\varepsilon}}{n_c^{1+\varepsilon} + n_h^{1+\varepsilon}}\hat{n}_{c,t} + \frac{n_h^{1+\varepsilon}}{n_c^{1+\varepsilon} + n_h^{1+\varepsilon}}\hat{n}_{h,t}\right) \quad (56)$$

Marginal disutility of working of the patient household in the housing sector:

$$\hat{u}_{nh,t} = \hat{z}_t + \hat{\tau}_t + \xi \hat{n}_{h,t} + (\eta - \varepsilon)\left(\frac{n_c^{1+\varepsilon}}{n_c^{1+\varepsilon} + n_h^{1+\varepsilon}}\hat{n}_{c,t} + \frac{n_h^{1+\varepsilon}}{n_c^{1+\varepsilon} + n_h^{1+\varepsilon}}\hat{n}_{h,t}\right) \quad (57)$$

Marginal utility of consumption of the impatient household:

$$(G_C - \varepsilon')(1 - \beta'\varepsilon')\hat{u}_{c',t} = (G_C - \varepsilon')(\hat{z}_t - \beta'\varepsilon'\hat{z}_{t+1}) - \left(G_C + \beta'\varepsilon'^2\right)\hat{c}'_t + \varepsilon'\hat{c}'_{t-1} + \beta'\varepsilon' G_C \hat{c}'_{t+1} \quad (58)$$

Marginal utility of housing of the impatient household:

$$\hat{u}_{h',t} = \hat{z}_t + \hat{j}_t - \hat{h}'_t \quad (59)$$

Marginal disutility of working of the impatient household in the consumption sector:

$$\hat{u}_{nc',t} = \hat{z}_t + \hat{\tau}_t + \xi' \hat{n}'_{c,t} + (\eta' - \varepsilon')\left(\frac{(n'_c)^{1+\varepsilon}}{(n'_c)^{1+\varepsilon} + (n'_h)^{1+\varepsilon}}\hat{n}_{c,t} + \frac{(n'_h)^{1+\varepsilon}}{(n'_c)^{1+\varepsilon} + (n'_h)^{1+\varepsilon}}\hat{n}'_{h,t}\right) \quad (60)$$

Marginal disutility of working of the impatient household in the housing sector:

$$\hat{u}_{nh',t} = \hat{z}_t + \hat{\tau}_t + \xi' \hat{n}'_{h,t} + (\eta' - \varepsilon')\left(\frac{(n'_c)^{1+\varepsilon}}{(n'_c)^{1+\varepsilon} + (n'_h)^{1+\varepsilon}}\hat{n}_{c,t} + \frac{(n'_h)^{1+\varepsilon}}{(n'_c)^{1+\varepsilon} + (n'_h)^{1+\varepsilon}}\hat{n}'_{h,t}\right) \quad (61)$$

GDP:

$$G\hat{D}P_t = \frac{\tilde{C}}{G\tilde{D}P}\hat{C}_t + \frac{\tilde{q}\tilde{H}}{G\tilde{D}P}\hat{H}_t + \frac{\tilde{I}_k}{G\tilde{D}P}\hat{I}_{k,t} \quad (62)$$

Total Investment:

$$\hat{I}_{k,t} = \frac{\tilde{I}_{k,c}}{\tilde{I}_k}\left(\hat{I}_{kc,t} - \hat{a}_{k,t}\right) + \frac{\tilde{I}_{k,h}}{\tilde{I}_k}\hat{I}_{kh,t} \quad (63)$$

Total Consumption:

$$\hat{C}_t = \frac{\tilde{c}}{\tilde{C}}\hat{c}_t + \frac{\tilde{c}'}{\tilde{C}}\hat{c}'_t \quad (64)$$

Investment in the Consumption sector:

$$\hat{I}_{kc,t} = \frac{\tilde{k}_c}{\tilde{I}_{kc}}\hat{k}_{c,t} + \frac{(1 - \delta_{kc})\tilde{k}_c}{G_{KC}\tilde{I}_{kc}}\hat{k}_{c,t-1} \quad (65)$$

Investment in the Housing sector:

$$\hat{I}_{kh,t} = \frac{\tilde{k}_h}{\tilde{I}_{kh}}\hat{k}_{h,t} + \frac{(1-\delta_{kh})\,\tilde{k}_h}{G_{KH}\tilde{I}_{kh}}\hat{k}_{h,t-1} \tag{66}$$

Exogenous component of the investment-specific technology shocks:

$$\hat{a}_{k,t} = \hat{Z}_{k,t} \tag{67}$$

A.1.3 Exogenous Variables

Intertemporal preference:

$$\hat{z}_t = \rho_z \hat{z}_{t-1} + u_{z,t} \tag{68}$$

Labor supply:

$$\hat{\tau}_t = \rho_\tau \hat{\tau}_{t-1} + u_{\tau,t} \tag{69}$$

Housing preferenc:

$$\ln \hat{j}_t = \rho_j \hat{j}_{t-1} + u_{j,t} \tag{70}$$

Collateral:

$$\hat{m}_t = \rho_m \hat{m}_{t-1} + u_{m,t} \tag{71}$$

Exogenous component in the investment-specific technology:

$$\hat{Z}_{k,t} = \rho_{AK} \hat{Z}_{k,t-1} + u_{K,t} \tag{72}$$

Exogenous component in the productivity of the consumption sector:

$$\hat{Z}_{c,t} = \rho_{AC} \hat{Z}_{c,t-1} + u_{C,t} \tag{73}$$

Exogenous component in the productivity of the housing sector:

$$\hat{Z}_{h,t} = \rho_{AH} \hat{Z}_{h,t-1} + u_{H,t} \tag{74}$$

Persistent monetary:

$$\hat{s}_t = \rho_s \hat{s}_{t-1} + u_{s,t} \tag{75}$$